

REPUBLIC OF TURKEY
YILDIZ TECHNICAL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

NUCLEAR PHYSICS IN NEUTRON STARS: STUDY OF
SUPERFLUIDITY IN HYPERNUCLEI AND
CONSTRAINING THE NUCLEAR EQUATION OF STATE

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A thesis submitted by Haşim Zahid Güven in partial fulfillment of the requirements for the degree of **DOCTOR OF PHILOSOPHY** is approved by the committee on 13.07.2020 in Department of Physics, Program of Physics.

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Dedicated to my family...

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LIST OF SYMBOLS

k	Angular Momentum Wave Vector
E_{HF}	Binding Energy for Hartree-Fock
E_{HFB}	Binding Energy for Hartree-Fock-Bogoliubov
$\tilde{\Lambda}$	Combined Tidal Deformability
E_{cond}	Condensation Energy
n_c	Critical Density
ϵ	Energy Functional (Chap. 2 and Chap. 3)
ϵ	Energy (Chap. 4 and Chap. 5)
w_{filter}	Filter Function
G	Gravitational Constant
M_c	Incompressibility at Critical Density
M_0	Incompressibility at Saturation Density
q	Isospin
δ	Isospin Asymmetry Parameter
n_0	Isoscalar Density
n_1	Isovector Density
T	Kinetic Energy (Chap. 4)
τ	Kinetic Energy Density (Chap. 2)
t	Kinetic Energy Density (Chap. 4)
$\hat{T}$	Kinetic Energy Operator
Λ	Lambda Hyperon (Chap. 2 and Chap. 3)
$P(\text{data} \{a_i\})$	Likelihood Function
m	Mass

M	Mass of Neutron Star
ρ	Matter Density (Chap. 4 and Chap. 5)
n_n	Neutron Density
ρ_{ND}	Neutronization Density
ρ	Normal (Particle) Density (Chap. 2 and Chap. 3)
$M_{\text{max}}^{\text{obs}}$	Observational Maximum Mass of Neutron Stars
$P(a_j \mid \text{data})$	One-Parameter Probability Distribution Function
$\tilde{\rho}$	Pairing Density
$\mathbf{r}$	Position (Coordinate Space)
$P(\{a_i\} \mid \text{data})$	Posterior Probability Distribution Function
P	Pressure
$P(2n_{\text{sat}})$	Pressure for Twice Saturation Density
$P(\{a_i\})$	Prior Probability Distribution Function
$p_{\tilde{\Lambda}}$	Probability Function Associated with GW170817
$p_{\chi^{\text{EFT}}}$	Probability Function Associated with χ^{EFT}
p_{ISGMR}	Probability Function Associated with <i>ISGMR</i>
n_p	Proton Density
k_2	Quadruple (Tidal) Love Number
R	Radius of Neutron Star
$R_{1.4}$	Radius of Neutron Star for $1.4M_{\odot}$
$\hbar$	Reduced Plank Constant
ρ_{sat}	Saturation Density (Matter Density)
n_{sat}	Saturation Density (Number Density)
$M_{\odot}$	Solar Mass
c	Speed of Light
σ	Spin
$C_{s,\infty}^2$	Square of the Speed of sound at Conformal Limit
T	Temperature (Chap. 1)
Λ	Tidal Deformability (Chap. 4 and Chap. 5)

$\Lambda_{1.4}$	Tidal Deformability for $1.4M_{\odot}$
$\widehat{H}$	Total Hamiltonian
T	Total Isospin (Chap. 3)
$P(a_j, a_k \mid$ data)	Two-Parameter Probability Distribution Function
ψ	Wave Function

LIST OF ABBREVIATIONS

AM	Asymmetric Matter
χEFT	Chiral Effective Field Theory
EoS	Equation of State
ESC	Extendend Soft Core Potentials
FG	Fermi Gas
GW	Gravitational Wave
HF	Hartree-Fock
HFB	Hartree-Fock-Bogoliubov or Bogoliubov-de Gennes
ISGMR	Isoscalar Giant Monopole Resonance
NM	Neutron Matter
NS	Neutron Star
NSC	Nijmegen Soft Core Potentials
PDF	Probability Distribution Function
RMF	Relativistic Mean Field
SHF	Skyrme-Hartree-Fock
SM	Symmetric Matter
TOV	Tolman-Oppenheimer-Volkoff
UG	Unitary Gas

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Nuclear Physics in Neutron Stars: Study of Superfluidity in Hypernuclei and Constraining the Nuclear Equation of State

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Department of Physics
Doctor of Philosophy Thesis

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In this thesis, we first investigated the effect of Λ pairing on the ground state properties of hypernuclei within the Hartree-Fock-Bogoliubov formalism. The SLy5 Skyrme functional is used in the NN channel, while for $N\Lambda$ channel we employ three functionals fitted from microscopic Brueckner-Hartree-Fock calculations: DF-NSC89, DF-NSC97a and DF-NSC97f. These functionals reproduce the sequence of single- Λ experimental binding energies from light to heavy hypernuclei. For the $\Lambda\Lambda$ channel, we used the empirical prescription EmpC, calibrated to 1 MeV on the experimental bond energy in ${}^6\text{He}_{\Lambda\Lambda}$. Based on this density-functional approach, several nuclei have been studied with nucleon closed-shells and Λ open-shells. A $\Lambda\Lambda$ pairing interaction is introduced, which magnitude is calibrated to be consistent with the maximum BCS predictions for the Λ pairing gap in hypernuclear matter. In this way, we provide an upper bound for the prediction of the Λ pairing gap and its effects in hypernuclei. We have shown that the effects of the $\Lambda\Lambda$ pairing depends on hypernuclei. The condensation energy is predicted to be about 3 MeV as a maximum value, yielding small corrections on density distributions and shell structure. Generally, we found that $\Lambda\Lambda$ pairing could be active if the energy gap between shells is smaller than 3 MeV. Under this condition, Λ pairing could impact densities and binding energies. Since only a weak spin-orbit interaction is expected in the Λ channel, Λ states are highly degenerated and usually levels are distant by more than 3 MeV in energy. In summary, it is shown that the Λ -related pairing effect can usually be neglected in

most of hypernuclei, except for hypernuclei which have a single particle gap lower than 3 MeV around the Fermi level. In addition, conditions on both Fermi energies and orbital angular momenta are expected to quench the nucleon- Λ pairing for most of hypernuclei.

The second part of the thesis is devoted to equation of states in neutron stars. We confronted the tidal deformability values extracted from the gravitational event GW170817 to nuclear physics constraints within a semi-agnostic approach for the dense matter equation of state. We used Bayesian statistics to combine together low density nuclear physics data, such as the ab-initio predictions based on χ EFT interactions or the isoscalar giant monopole resonance, and astrophysical constraints from neutron stars, such as the maximum mass of neutron stars or the probability density function of the tidal deformability $\tilde{\Lambda}$ obtained from the GW170817 event. The posteriors probability distribution functions are marginalized over several nuclear empirical parameters (L_{sym} , K_{sym} , Q_{sat} and Q_{sym}), as well as over observational quantities such as the $1.4M_{\odot}$ radius $R_{1.4}$ and the pressure at twice the saturation density $P(2n_{\text{sat}})$. The correlations between L_{sym} and K_{sym} and between K_{sat} and Q_{sat} are also further analyzed. It is found that there is a marked tension between the gravitational wave observational data and the nuclear physics inputs for the L_{sym} and $R_{1.4}$ marginal probability distributions. This could be a hint for nucleons to more exotic particles phase transition inside of the core of neutron stars. We also conclude that increasing the accuracy on the determination of tidal deformability from the gravitational wave, as well as M_c from the isoscalar giant monopole resonance, will lead to a better determination of K_{sat} and Q_{sat} .

Keywords: Neutron Stars, Hypernuclei, Equation of State, Gravitational Wave, Nuclear Structure, Superfluidity

Nötron Yıldızlarında Nükleer Fizik: Hiperçekirdeklerde Süperakışkanlık Çalışması ve Nükleer Durum Denkleminin Kısıtlanması

Haşim Zahid Güven

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Bu tezde öncelikle Λ çiftleniminin hiperçekirdeklerin taban durum özelliklerine etkisi Hartree-Fock-Bogoliubov yaklaşımı ile incelenmiştir. NN kanalı için SLy5 Skyrme tipi kuvvet fonksiyoneli ve $N \Lambda$ kanalı için Brueckner-Hartree-Fock hesaplamalarından elde edilmiş 3 tip kuvvet seti kullanılmıştır: DF-NSC89, DF-NSC97a ve DF-NSC97f. Bu fonksiyoneller hafiften ağıra, bütün tek- Λ 'lı hiperçekirdeklerin deneysel bağlanma enerjilerini yüksek doğrulukta yeniden üretmektedir. $\Lambda\Lambda$ kanalı için, ${}^6\text{He}_{\Lambda\Lambda}$ hiperçekirdeğinin deneysel bağ enerjisi olan 1 MeV değerini hesaplayan ampirik EmpC kuvvet seti kullanılmıştır. Yoğunluk fonksiyonel yaklaşımı ile, kapalı ve açık kabuk pek çok hiperçekirdek bu tez içerisinde hesaplanmıştır. Açık kabuk hiperçekirdekler için $\Lambda\Lambda$ çiftlenim etkileşimi hipernükleer madde hesaplarındaki BCS yaklaşımına dayanılarak tanımlanmıştır. Böylece hiperçekirdeklerdeki çiftlenim enerjisinin üst limiti ve çiftlenim etkileşmesinin hiperçekirdeklerin taban durum özelliklerine etkisi hesaplanmıştır. Bu çalışma sonucunda $\Lambda\Lambda$ çiftlenim etkileşmesinin hiperçekirdekten hiperçekirdeğe değişmesi ile beraber, çiftlenim etkisinden kaynaklı yoğunlaşma enerjisinin maksimum 3 MeV olduğu hesaplanmıştır. Ayrıca çiftlenim etkileşmesinin, nükleon ya da hiperon yoğunluk dağılımlarına ve kabuk yapısına sadece küçük düzeltmeler yaptığı ortaya çıkarılmıştır. Genellikle boş-dolu kabuklar arasındaki enerji farkı 3 MeV değerinden düşükse çiftlenim etkisinin ortaya çıktığı ve bu durumda hiperçekirdeğin yoğunluk dağılımlarını ve bağlanma enerjilerini değiştirdiği ortaya çıkarılmıştır. Ancak hiperçekirdeklerde spin-yörünge etkileşmesi

çok küçük olduğu ve ihmal edildiğinden ötürü kabuklar arasındaki enerji farkı yoğunlukla 3 MeV değerinden fazladır. Dolayısı ile Λ çiftlenim etkisinin pek çok hiperçekirdek için ihmal edilebilir olduğu görülmüştür. Buna ek olarak nükleon- Λ çiftleniminin, Fermi enerjileri ve kabukların açısal momentumu arasındaki farklardan ötürü yoğunlukla sönümlendiği gösterilmiştir. Tezin ikinci kısmı nötron yıldızlarının durum denkleminin incelenmesine ayrılmıştır. Bu amaçla GW170817 kütle-çekim dalgası kaynaklı gel-git deformasyonu verilerini kullanarak, nükleer durum denklemleri hesaplanmıştır. Böylece nükleer fizikten elde edilen sınırlar ile kütle-çekim dalgasının tanımladığı sınırlar karşılaştırılarak, nükleer fizik ile gözlemsel kütle-çekim dalgasının işaret ettiği verileri arasındaki fark tartışılmıştır. Durum denklemleri, yarı-agnostik yaklaşım kullanılarak oluşturulmuş, Bayesian istatistik kullanılarak nötron yıldızının durum denklemi, nükleer fizikten bilinen ve astrofiziksel gözlemlerden elde edilen veriler ile sınırlandırılmıştır. Ab-initio χ EFT kullanılarak oluşturulan nükleer fizik verisi ve izoskalar dev monopol rezonansı nükleer fizik kaynaklı sınırlayıcılar olarak kullanılmıştır. Nötron yıldızlarının maksimum kütlesi ve kütle çekim dalgası kaynaklı gel-git deformasyonu $\tilde{\Lambda}$ ise astrofizik kaynaklı sınırlayıcılar olarak kullanılmıştır. Bunun sonucu olarak posterior olasılık dağılım fonksiyonları nükleer ampirik parametreler üzerinden marjinalize edilerek (L_{sym} , K_{sym} , Q_{sat} and Q_{sym}), her bir nükleer ampirik parametrenin olasılık dağılımları hesaplanmıştır. Bununla beraber kanonik nötron yıldızı kütlesi ($1.4M_{\odot}$) için yarı çap dağılımı $R_{1.4}$ ve nükleer doyum yoğunluğunun 2 katında varolan basınç $P(2n_{\text{sat}})$ gibi gözlemsel verilerde bu tez içerisinde ayrıca hesaplanmıştır. Ayrıca L_{sym} - K_{sym} ve K_{sat} - Q_{sat} nükleer ampirik parametreleri arasındaki korelasyonlar da bu tez içerisinde gösterilmiştir. Bütün bunların akabinde nükleer fizik verileri ile kütle çekim dalgası verileri arasında özellikle L_{sym} ve $R_{1.4}$ için çok büyük uyumsuzluklar gözlemlenmiştir. Bu uyumsuzluk nötron yıldızı çekirdeğinde, maddenin daha egzotik bir haline doğru faz değişimine işaret edebilir. Ayrıca kütle-çekim dalgasının ve dev izoskalar monopol rezonans kaynaklı M_c değerinin ölçümsel doğruluğunun arttırılması, K_{sat} ve Q_{sat} parametrelerinin daha iyi belirlenmesinde rol oynayacağı sonucuna vardık.

Anahtar Kelimeler: Nötron Yıldızları, Hiperçekirdekler, Durum Denklemi, Kütle-Çekim Dalgası, Nükleer Yapı, Süperakışkanlık

1

INTRODUCTION

1.1 Literature Review

One of the most important quests of nuclear physics is the finding an universal interaction for the whole baryon family. In order to pursue this quest, more than 4000 scattering experiments have been performed until now [1]. The scattering data tightly constrains our models of the nucleon–nucleon interaction. However, while these experiments for finite nuclei probe a range of densities around saturation density of nuclear matter ($n_{\text{sat}} \approx 0.16 \text{ fm}^{-3}$, $\rho_{\text{sat}} \approx 2.7 \times 10^{14} \text{ g/cm}^3$) and heavy-ion collisions explore a wider domain of densities with small isospin asymmetries, neutron stars (NSs) are the only system (for now) which explores the equilibrium properties of dense matter at densities well above saturation density and isospin asymmetries close to pure neutron matter [2]. NS physics addresses thus one of the most fundamental questions in nuclear physics which is the understanding of the nuclear interaction in dense medium as a function of the density and the isospin asymmetry. They are excellent systems where the high density behavior of the nuclear equation of state (EoS) can potentially be determined. Although there are considerable astrophysical observations, a lot of uncertainties related to the structure of NS still exist.

Let us first give some information about the structure of NS. The outermost surface of the NS contains a very thin atmosphere of only a few centimeters thick that is composed of H, but may also contain heavier elements such as He and C [3]. The detected electromagnetic radiation may be used to constrain critical parameters of the neutron star. For instance black-body emission from the stellar surface at given temperature provides a determination of the stellar radius from the Stefan-Boltzmann law. Unfortunately there are lots of uncertainties to determine exact values of radii (see Sec. 4.1 for details). Just below the atmosphere, the 1 km thick envelope (which also named crust) behave as a blanket between the cold atmosphere (with $T = 10^6 \text{ K}$) and hot core (with $T = 10^8 \text{ K}$) [3, 4]. The crust density varies from $10^{-3}\rho_{\text{sat}}$ to $0.5\rho_{\text{sat}}$ [3]. It consists of electrons, free neutrons, and neutron-rich atomic nuclei. However, free neutrons start to appear where the density higher than the

neutronization density ($\rho_{\text{ND}} = 4 \times 10^{11} \text{ g/cm}^3$). This region usually named inner crust. As the density grows, the fraction of free neutrons increase [3]. Besides, free neutrons and neutron rich-nuclei can also be in a superfluid state depending on temperature which has a significant impact on cooling scenarios of NSs [4, 5]. At a density larger than $0.5\rho_{\text{sat}}$, the finite-nuclear phase disappears and a new state of matter is formed [3]. This section of NS is called "core" which is responsible for all the mass of NS and approximately 95% of its total radius. Besides, its density could even reach up to $10\rho_{\text{sat}}$ [6]. However, this reality comes with its own problems since the behavior of the EoS for densities $\rho \geq \rho_{\text{sat}}$ is not accurately known from experiments of nuclear physics and also their extrapolations to $2 - 10\rho_{\text{sat}}$ are not solely reliable. For convenience, a general nomenclature is created which is named "outer core" for densities $0.5\rho_{\text{sat}} < \rho \leq 2\rho_{\text{sat}}$ and "inner core" for densities $\rho \geq 2\rho_{\text{sat}}$ [3]. The EoS of NS for outer core can be investigated by using nucleonic models derived from the nuclear physics experiments since these experiments represent the knowledge around the saturation density. In this case, NS can be considered as nuclear matter consisting of neutrons and protons, as well as a gas of electrons and muons at β equilibrium [2, 3, 7, 8]: This is the traditional description of the NS. However for densities $\rho \geq 2\rho_{\text{sat}}$, the EoS of NS is completely unknown. Therefore, three main hypothesis are proposed to explain the inner core of NS:

- Phase transition to hypernuclear matter: Appearance of hyperons on some onset density (typically $\rho > 2\rho_{\text{sat}}$), specifically Σ^- and Λ hyperons [9–13].
- Phase transition to quark matter: Deconfined light u and d quarks and strange s quarks, and a small admixture of electrons (or not) could form a true ground state of matter which is also called strange matter hypothesis [14]. However, it is a debated issue and there can be a two different scenarios leading to either the whole star could be quark matter which is also called as a quark star or the quark phase transition could occur only at a sufficiently high density [15–18]. In the case of quark phase transition, building core of NS by using quarks results in significantly small radii compared to the traditional ones [15–18] except for the quarkyonic model, which describes the transition to quark matter as a crossover [19].
- Pion condensation: The appearance of a boson condensate of pion-like excitations with a strong renormalization and mixing of nucleon states [20–22]. This hypothesis is considered as the least likely, since the effect of pion condensation on EoS is predicted to be negligible from nuclear matter calculations [21].

1.2 Objective of the Thesis

Considering the first hypothesis, at large densities, a substantial population of hyperons are expected because the Fermi energy of neutrons becomes of the order of their rest mass, leading to an increase of the hyperon fraction, but it also reduces the degeneracy pressure inside the NS, leading to soft NS EoS at high densities, causing a problem to reach maximum observed NS mass: $2M_{\odot}$ [23, 24] where $M_{\odot}$ is the solar mass. This problem could be alleviated by adding the hypothetical repulsive interaction using some vector mesons, specifically the ϕ meson (see Ref. [15] for details.) Therefore, extracting the hyperon interaction inside the nuclear medium from hypernuclear experiments has an utmost importance. Besides, for the case of multi-strange hypernuclei, there could be superfluidity (or in simple terms: pairing) which can affect the interpretation of the experiments related to multi-strange hypernuclei. However, the Λ pairing channel in hypernuclei is completely unknown. Besides, the pairing interaction in the strange sector could have an important effect on the cooling curves of neutron stars. Therefore, the first part of the thesis is devoted to the investigation of the Λ pairing channel on hypernuclei.

The advent of first detection of gravitational waves from a binary NS merger (GW170817) by the LIGO-Virgo collaboration [25, 26], opens a new era for nuclear astrophysics since it provides an additional observable related to the EoS of NS, among which the Tidal deformability ($\tilde{\Lambda}$) [27–29]. Considering this "new" observable, the second part of the thesis is devoted to constrain the nuclear EoS. To do this, nuclear EoSs generated by using observational data such as the maximum mass ($2M_{\odot}$ see Refs. [23, 24] for details) and $\tilde{\Lambda}$ from the gravitational wave event of GW170817 [25, 26, 30, 31] as well as predictions from nuclear physics such as Chiral Effective Field Theory (χ EFT) [32] and Isoscalar Giant Monopole Resonance (ISGMR) [33, 34] are confronted to each other.

1.3 Hypothesis

In Chap. 2, the theoretical foundations of mean field approximation (with Hartree-Fock-Bogoliubov or Bogoliubov-de Gennes equations) for hypernuclei are explained [35, 36]. We considered hypernuclei with proton and neutron closed shells, e.g. $^{40}_{-\Lambda}\text{Ca}$, $^{132}_{-\Lambda}\text{Sn}$ and $^{208}_{-\Lambda}\text{Pb}$, since semi-magicity often guarantees that nuclei remain at, or close to, sphericity. In Chap. 3, both nucleon- Λ and Λ - Λ pairing channel are investigated and their effect on the ground state properties such as binding energies, single particle spectrums and particle densities are calculated on $^{40}_{-\Lambda}\text{Ca}$, $^{132}_{-\Lambda}\text{Sn}$ and $^{208}_{-\Lambda}\text{Pb}$ hypernuclei.

In Chap. 4, the theoretical fundamentals of meta-model [37], Tolman-Oppenheimer-Volkoff (TOV) [38, 39] and Pulsation equations [27–29] are given. In order to connect observational data with theoretical predictions of nuclear physics, a brief overview on Bayesian statistics is also given [40]. Finally with the power of Bayesian statistics, the building of the posterior probability from the likelihood one, which includes all constraints, and from the prior on the model parameters, are detailed. In Chap. 5, the analysis of the posterior probabilities is undertaken for the following empirical parameters: L_{sym} , K_{sym} , Q_{sat} and Q_{sym} as well as for the radius of $1.4M_{\odot}$, $R_{1.4}$, and the pressure at $2n_{\text{sat}}$, $P(2n_{\text{sat}})$. We then analyze origins of the correlation between the L_{sym} - K_{sym} and K_{sat} - Q_{sat} parameters. Finally, their implications on nuclear and NS physics are discussed in detail.

PAIRING IN HYPERNUCLEI: THEORETICAL FOUNDATIONS

2.1 A Strange System: Hypernucleus and Hypernuclei

A hypernucleus is an extra-ordinary nucleus including ordinary nucleons with one (or more) strange baryons (hyperons). It was first detected from hyper-fragments exposed to cosmic rays [41] in 1952. One year after, a new quantum number, "strangeness", was introduced [42, 43]. The reason why it is called "strangeness" is that, these systems are bound with the time-scale of strong interaction (10^{-23} s) but decay only with the weak interaction (which time-scale is 10^{-10} s) inside of the nuclear medium. Therefore, hypernuclei can be investigated with a many-body framework typically used in nuclear physics. Within the last 40 years, modern particle accelerators and electronic instrumentation have increased the rate and breadth of the experimental investigation of strangeness in nuclei (especially, the Japan Proton Accelerator Research Complex in Japan and the proton antiproton detector array at GSI Facility for Antiproton and Ion Research [44–47]). As often, the theoretical interest has closely followed the experimental development.

A hyperon is characterized by its strangeness number S which is $S = -1$ for Λ , Σ^+ , Σ^- , Σ^0 and $S = -2$ for Ξ^- , Ξ^0 . The strangeness of a hypernucleus is determined by the strangeness number of the hyperon if the hypernucleus contains only one hyperon. However, some hypernuclei can contain more than one hyperon, forming a multi-strangeness hypernucleus where the strangeness of the system is defined by the total number of each hyperon.

Although in principle, any hyperon could bound with an ordinary nucleus to form a hypernucleus, there is indeed a specific attention for the Λ since the mass ($m_\Lambda = 1115.683 \pm 0.006$ MeV/ c^2) is the lightest one compared to other hyperons ($m_{\Sigma^+} = 1189.370 \pm 0.070$ MeV/ c^2 , $m_{\Sigma^0} = 1192.642 \pm 0.024$ MeV/ c^2 , $m_{\Sigma^-} = 1197.449 \pm 0.030$ MeV/ c^2 , $m_{\Xi^-} = 1321.710 \pm 0.070$ MeV/ c^2 and $m_{\Xi^0} = 1314.860 \pm 0.200$ MeV/ c^2) [48]. It is therefore easier to produce Λ than other hyperons. Another

reason for choosing Λ is that since other hyperons (Σ^+ , Σ^- , Σ^0 , Ξ^- and Ξ^0) decay to Λ inside the nuclear medium, observing Λ hypernuclei is the easiest way to understand hyperon-nucleon interaction at first order [49]. As a consequence, there are sufficient experimental data related to single- Λ hypernuclei, from ${}^5_{\Lambda}\text{He}$ to ${}^{208}_{\Lambda}\text{Pb}$, which are relevant to understand the Λ -nucleon interaction, at least at very low density. Λ separation energies and single particle structures of these hypernuclei is now fairly known [50, 51]. However, for Σ hypernuclei, it is generally assumed that the bound Σ hypernuclear system does not exist except for ${}^4_{\Sigma}\text{He}$ [52, 53]. The Σ^+ separation energy for ${}^4_{\Sigma}\text{He}$ was measured to be 3.2 MeV [52] (for comparison: 2.39 MeV for ${}^4_{\Lambda}\text{He}$ [50, 51]). Besides, Ref. [53] shows that ${}^4_{\Sigma}\text{He}$ has not any excited state. In the $-S \geq 2$, the situation is slightly better than the Σ case. There are few experimental data about double- Λ hypernuclei due to their low production rates of kaons, with some observed hypernuclei such as ${}^6_{\Lambda\Lambda}\text{He}$ or ${}^{10}_{\Lambda\Lambda}\text{Be}$, allowing to extract of the bond energy which is strongly related to Λ - Λ interaction [45, 46, 54, 55]. The famous "NAGARA" event allows to determine the bond energy of ${}^6_{\Lambda\Lambda}\text{He}$ to be 0.6-1 MeV [45, 46, 54, 55]. Another case for $-S = 2$ is the Ξ hypernucleus. Unfortunately, there is only one event reported related to Ξ hypernucleus, "KISO" event for ${}^{15}_{\Xi^-}\text{C}$ [56] where Ξ^- separation energies allow to conclude either $B_{\Xi^-} = 4.38 \pm 0.25$ MeV or $B_{\Xi^-} = 1.11 \pm 0.25$ MeV (see Ref. [1] for details). However, if one considers the coulomb interaction which is predicted to be 3 MeV for ${}^{15}_{\Xi^-}\text{C}$ [1], Ξ -nucleon interaction could be either attractive or repulsive. All in all, due to the large experimental uncertainty on Σ and Ξ hypernuclei measurements, in the present work, we consider the case of Λ hypernuclei.

Let us now discuss about the theoretical description of hypernuclei. Λ hypernuclei have often been considered as the best system to investigate the nature of hyper-nuclear interactions in the baryon octet [51, 57]. Despite the numerous theoretical works about hypernuclei physics within various frameworks, such as relativistic mean field [58–61], G-matrix combined with Skyrme-Hartree-Fock for finite-nuclei [62–65], generalized liquid drop model [66], as well as more recently quantum Monte-Carlo approach [67, 68], there are still open questions concerning the understanding of multi-strange nuclei and the hypernuclear equation of state. In general the main difficulties for theoretical approaches is the very scarce amount of experimental data, as explained in the previous paragraph. Constraints on the hyperon interactions are therefore still weak. As an example, the NNA interaction is still subject of debate [67, 68]. Most of the recent theoretical approaches predict binding energies and single particle energies of single- Λ systems such as ${}^5_{\Lambda}\text{He}$, ${}^9_{\Lambda}\text{Be}$, ${}^{13}_{\Lambda}\text{C}$, ${}^{209}_{\Lambda}\text{Pb}$ in good agreement with the experimental data [58, 59]. In the present work, for instance, we consider density functional approaches where the nucleon sector is treated with Skyrme interaction and the NA channel is based on G-matrix calculations starting from

various bare interactions such as NSC89, NSC97a–f (Nijmegen Soft Core Potentials) or ESC08 (Extendend Soft Core Potentials) [63, 65]. The older DF-NSC89 functional can reproduce with a good accuracy the experimental single particle energies of Λ hyperon for light hypernuclei such as ${}^5_{\Lambda}\text{He}$ or ${}^{13}_{\Lambda}\text{C}$, but for heavier hypernuclei like ${}^{41}_{\Lambda}\text{Ca}$ or ${}^{209}_{\Lambda}\text{Pb}$, DF-NSC97a–f and DF-ESC08 are better [63, 65]. It should be noted that this discrepancy between the interactions can be removed with adequate fitting such as adding new terms in the energy functional for the single Λ hypernuclei (see Ref. [63] for details).

However, there are still many open questions related to hypernuclei. The first one deals with the strength of the $\Lambda\Lambda$ force. In general, the experimental bond energy of multi-strange systems, such as ${}^6_{\Lambda\Lambda}\text{He}$ or ${}^{10}_{\Lambda\Lambda}\text{Be}$, are not reproduced by the usual density functionals [61]: the DF-NSC89 and DF-NSC97f forces predict bond energies ranging from -0.34 MeV (${}^{10}_{\Lambda\Lambda}\text{Be}$) to -0.12 MeV (${}^{210}_{\Lambda\Lambda}\text{Pb}$) while the DF-NSC97a, which has strong $\Lambda\Lambda$ interaction, predicts bond energies from 0.37 MeV (${}^{10}_{\Lambda\Lambda}\text{Be}$) to 0.01 MeV (${}^{210}_{\Lambda\Lambda}\text{Pb}$) [65]. It should be noted that more recent density functionals, e.g. the one derived from the ESC08 potential, do not better to reproduce the bond energy (-0.57 MeV for ESC08 in ${}^6_{\Lambda\Lambda}\text{He}$ [65]). In order to improve the description of the $\Lambda\Lambda$ force, an empirical correction for DF-NSC89 and DF-NSC97a–f in the $\Lambda\Lambda$ channel has been proposed and fitted to the bond energies of ${}^6_{\Lambda\Lambda}\text{He}$ (which is ~ 0.6 - 1 MeV [45, 46, 54, 55]) and named EmpC (see Ref. [69] for details). Note that similar issues exist with relativistic approaches for hypernuclei and it was recently proposed that the fit of the bond energy shall enter directly in the definition of the Lagrangian [12]. Therefore we shall use DF-NSC89+EmpC, DF-NSC97a+EmpC and DF-NSC97f+EmpC potentials, due to the compatible results of $\Lambda\Lambda$ channel.

Besides, a well-known issue is indeed the so-called hyperon puzzle in neutron stars [9–13, 15]. It refers to the difficulty for many approaches to reach the largest observed mass of neutron stars, of about $1.9 - 2.0 M_{\odot}$ [23, 70] when considering the softening induced by the onset of hyperons in the nuclear matter. The possible solutions may be that the hyperon interaction turns strongly repulsive in dense matter, counterbalancing the softening of the phase transition to hypernuclear matter [9–13, 15], or quark phase appears before the hypernuclear matter [16–19]. A detailed discussion about the phase transition considering hyperons (or a quark phase) on neutron stars is given in Sec. 4.2.

There is however a question which has not been addressed yet and may modify our understanding of the $\Lambda\Lambda$ channel in finite hypernuclei: are Λ particles paired, and how much Λ pairing impacts the ground state properties of hypernuclei (density distributions, binding energies, etc.)? It should be noted that although there is

currently no microscopic calculation in hypernuclei including Λ pairing, the pairing gap in hypernuclear matter has been calculated within the Bardeen–Cooper–Schrieffer (BCS) approximation [71–75]. The present work aims to provide a first investigation to the Λ pairing in finite hypernuclei by considering a rather optimistic scenario for the strength of $\Lambda\Lambda$ pairing.

Another question related to multi-strange nuclei deals with the presence of other strange baryons than Λ , such as Ξ , Σ or Ω . Ξ is the most crucial one because the $\Lambda + \Lambda \rightarrow N + \Xi$ decay channel (also called Ξ -instability) can make it appear in the ground state of hypernuclei [69, 76]. The attractive nature of the Ξ potential in nuclear matter ($U_{\Xi} = -14$ MeV)[77] stabilizes Ξ and both Ξ^0 and Ξ^- hyperons are predicted in hypernuclei with a strangeness number $-S \geq 20 - 30$ [69]. Besides, a Λ hyperon can also decay to Σ^0 , Σ^+ and Σ^- . However, due to the high Q^{free} values of Σ hyperons ($Q_{\Sigma}^{free} = -80$ MeV for Σ and $Q_{\Xi}^{free} = -26$ MeV for Ξ [69, 76]), the decay of Λ to $\Sigma^{\pm,0}$ is not favored in finite hypernuclei. Since the presence of Ξ in the ground state is not expected to enhance the Λ pairing, and since the pairing in the Ξ channel is even less known than the one in the Λ channel, we shall focus our present study on the pairing in Λ hypernuclei. We do not expect our conclusions to be strongly modified by the presence of hyperons other than Λ in finite nuclei.

2.2 Main Principles

In the present work, the ground state properties of single and multi- Λ hypernuclei are investigated with Hartree-Fock-Bogoliubov (HFB) formalism considering $\Lambda\Lambda$ pairing interactions. On this purpose we neglect the Λ spin-orbit interaction, which is estimated to be very small [50, 78, 79] and the three body interactions such as $NN\Lambda$ [67, 68] is effectively included from the functional approach. We have considered a zero range pairing force in the $\Lambda\Lambda$ channel, opening the possibility to calculate accurately open- Λ shell hypernuclei. In addition, our calculation are performed in spherical symmetry since deformation is not expected to greatly increase pairing correlations. We have considered hypernuclei with proton and neutron closed shells, e.g. $^{40}_{-\Lambda}\text{Ca}$, $^{132}_{-\Lambda}\text{Sn}$ and $^{208}_{-\Lambda}\text{Pb}$, since semi-magicity often guarantees that nuclei remain at, or close to, sphericity (see Ref. [80] for the general case of $N = Z$ hypernuclei). A future study evaluating the effect of deformation on the pairing correlation for open shell Λ states would however be interesting to confirm our conclusions.

2.3 Mean Field Approximation

Considering a non-relativistic system composed of interacting nucleons $N=(p,n)$ and Λ 's, the total Hamiltonian reads,

$$\widehat{H} = \widehat{T}_N + \widehat{T}_\Lambda + \widehat{H}_{NN} + \widehat{H}_{N\Lambda} + \widehat{H}_{\Lambda\Lambda}, \quad (2.1)$$

where $\widehat{T}_A$ are the kinetic energy operators and $\widehat{H}_{AB}$ are the interaction operator terms acting between A and B species ($A, B = N, \Lambda$).

2.3.1 The Particle-Hole Channel

In the mean field approximation the ground state of the system is the tensor product $|\Psi_N\rangle \otimes |\Psi_\Lambda\rangle$, where $|\Psi_N\rangle$ ($|\Psi_\Lambda\rangle$) is a Slater determinant of the nucleon (Λ) states. The total Hamiltonian (2.1) can be turned into a density functional $\epsilon(\rho_N, \rho_\Lambda)$, of the particle densities ρ_N and ρ_Λ , as $\widehat{H} = \int \epsilon(\rho_N, \rho_\Lambda) d^3r$. The energy functional ϵ is often expressed as [65, 81],

$$\begin{aligned} \epsilon(\rho_N, \rho_\Lambda) = & \frac{\hbar}{2m_N} \tau_N + \frac{\hbar}{2m_\Lambda} \tau_\Lambda + \epsilon_{NN}(\rho_N) \\ & + \epsilon_{N\Lambda}(\rho_N, \rho_\Lambda) + \epsilon_{\Lambda\Lambda}(\rho_\Lambda), \end{aligned} \quad (2.2)$$

where τ_N (τ_Λ) is the nucleonic (Λ) kinetic energy density and ϵ_{ij} are the interaction terms of the energy density functional describing the NN, N Λ and $\Lambda\Lambda$ channels.

In the following, the nucleonic terms will be deduced from the well known SLy5 Skyrme interaction [82] widely used for the description of the structure of finite nuclei, while the $N\Lambda$ channel is given by a density functional $\epsilon_{N\Lambda}$ adjusted to Brueckner-Hartree-Fock (BHF) predictions in uniform matter [65, 81],

$$\epsilon_{N\Lambda}(\rho_N, \rho_\Lambda) = -f_1(\rho_N) \rho_N \rho_\Lambda + f_2(\rho_N) \rho_N \rho_\Lambda^{5/3}. \quad (2.3)$$

Since the spin-orbit doublets are experimentally undistinguishable [50, 78], the spin-orbit interaction among Λ particles is also neglected [83]. The following density functionals are considered for the $N\Lambda$ channel: DF-NSC89 from Ref. [81], DF-NSC97a and DF-NSC97f from Ref. [65].

In the $\Lambda\Lambda$ channel $\epsilon_{\Lambda\Lambda}$ is adjusted to the experimental bond energy in ${}^6_{\Lambda\Lambda}\text{He}$ [69] from Nagara event (see Sec. 2.1 for details) [45, 46, 54, 55]:

$$\epsilon_{\Lambda\Lambda}(\rho_\Lambda) = -f_3(\rho_\Lambda) \rho_\Lambda^2. \quad (2.4)$$

Table 2.1 Parameters of the functionals DF-NSC89, DF-NSC97a and DF-NSC97f considering EmpC prescription for α_7 [69]

Functional	α_1	α_2	α_3	α_4	α_5	α_6	α_7
DF-NSC89 +EmpC	327	1159	1163	335	1102	1660	22.81
DF-NSC97a +EmpC	423	1899	3795	577	4017	11061	21.12
DF-NSC97f +EmpC	384	1473	1933	635	1829	4100	33.25

The corresponding empirical approach EmpC for the $\Lambda\Lambda$ channel is considered in the present work (see Ref. [69] for details). The functions f_{1-3} in Eqs. (2.3, 2.4) are expressed as,

$$f_1(\rho_N) = \alpha_1 - \alpha_2 \rho_N + \alpha_3 \rho_N^2, \quad (2.5)$$

$$f_2(\rho_N) = \alpha_4 - \alpha_5 \rho_N + \alpha_6 \rho_N^2, \quad (2.6)$$

$$f_3(\rho_\Lambda) = \alpha_7 - \alpha_8 \rho_\Lambda + \alpha_9 \rho_\Lambda^2, \quad (2.7)$$

where α_{1-7} are constants given in Tab. 2.1. Since we completely don't know the high density behavior of hyperon-hyperon interaction, the parameters α_8 and α_9 are therefore simply chosen to be 0 (the same approach has been done in Ref. [69]).

In uniform nuclear matter the single particle energies read,

$$\epsilon_N(k) = \frac{\hbar^2 k^2}{2m_N^*} + v_{NN}^{matt.} \text{ and } \epsilon_\Lambda(k) = \frac{\hbar^2 k^2}{2m_\Lambda^*} + v_\Lambda^{matt.}, \quad (2.8)$$

where the $v_\Lambda^{matt.}$ decomposes as,

$$v_\Lambda^{matt.} = v_{N\Lambda}^{matt.} + v_{\Lambda\Lambda}^{matt.}. \quad (2.9)$$

The potentials $v_{NN}^{matt.}$, $v_{N\Lambda}^{matt.}$ and $v_{\Lambda\Lambda}^{matt.}$ derive from the energy functionals. Namely,

$$v_{NN}^{matt.}(\rho_N, \rho_\Lambda) = v_N^{Skyrme} + \frac{\partial \epsilon_{N\Lambda}}{\partial \rho_N}, \quad (2.10)$$

$$v_{N\Lambda}^{matt.}(\rho_\Lambda) = \frac{\partial \epsilon_{N\Lambda}}{\partial \rho_\Lambda}, \quad (2.11)$$

$$v_{\Lambda\Lambda}^{matt.}(\rho_\Lambda) = \frac{\partial \epsilon_{\Lambda\Lambda}}{\partial \rho_\Lambda}. \quad (2.12)$$

Table 2.2 The parameters of the Λ -effective mass

Force	μ_1	μ_2	μ_3	μ_4
DF-NSC89	1.00	1.83	5.33	6.07
DF-NSC97a	0.98	1.72	3.18	0
DF-NSC97f	0.93	2.19	3.89	0

The nucleon effective mass is given from Skyrme interaction [84] and the Λ effective mass is expressed as a polynomial in the nucleonic density ρ_N as [81],

$$\frac{m_\Lambda^*(\rho_N)}{m_\Lambda} = \mu_1 - \mu_2 \rho_N + \mu_3 \rho_N^2 - \mu_4 \rho_N^3. \quad (2.13)$$

The values for the parameters μ_{1-4} are given in Tab. 2.2.

In hypernuclei, energy densities ϵ_N and ϵ_Λ are corrected by the effective mass term before deriving potentials as (see Ref. [69] and therein),

$$\epsilon_{NN}^{nucl}(\rho_N) = \epsilon_{NN}(\rho_N) - \frac{3\hbar^2}{10m_N} \rho_N^{5/3} \left(\frac{6\pi^2}{g_N} \right)^{2/3} \left[\frac{m_N}{m_N^*} - 1 \right], \quad (2.14)$$

$$\epsilon_{N\Lambda}^{nucl}(\rho_N, \rho_\Lambda) = \epsilon_{N\Lambda}(\rho_N, \rho_\Lambda) - \frac{3\hbar^2}{10m_\Lambda} \rho_\Lambda^{5/3} \left(\frac{6\pi^2}{g_\Lambda} \right)^{2/3} \left[\frac{m_\Lambda}{m_\Lambda^*} - 1 \right], \quad (2.15)$$

$$\epsilon_{\Lambda\Lambda}^{nucl}(\rho_\Lambda) = \epsilon_{\Lambda\Lambda}(\rho_\Lambda), \quad (2.16)$$

where $\epsilon_{NN}^{nucl}(\rho_N)$ satisfies

$$\frac{\partial \epsilon_{NN}^{nucl}(\rho_N)}{\partial \rho_N} = v_N^{Skyrme}. \quad (2.17)$$

The present functional (SLy5 in the NN channel, DF-NSC in the N Λ channel and EmpC in the $\Lambda\Lambda$ channel) therefore yields an optimal set to perform HF calculations in hypernuclei (see [69] for details).

2.3.2 Hartree-Fock-Bogoliubov Equations

The Hartree-Fock-Bogoliubov (HFB) framework is well designed for the treatment of pairing both for strongly and weakly bound systems. In this work, we study hypernuclei which are magic in neutron and proton and open-shell in Λ . We thus consider the HFB framework in the $\Lambda\Lambda$ channel, and the NN channel is treated within Hartree-Fock (HF). In addition, because of their magic properties in the nucleon sector,

which still contains the majority of particles, we consider spherical symmetry. In the HFB approach the mean field matrix that characterizes the system is obtained from the particle and pairing energy densities [36]. Particle and pairing densities can be expressed as

$$\rho(\mathbf{r}\sigma q, \mathbf{r}'\sigma'q') = \langle \psi | a_{\mathbf{r}'\sigma'q'}^+ a_{\mathbf{r}\sigma q} | \psi \rangle, \quad (2.18)$$

$$\tilde{\rho}(\mathbf{r}\sigma q, \mathbf{r}'\sigma'q') = -2\sigma' \langle \psi | a_{\mathbf{r}'-\sigma'q'}^+ a_{\mathbf{r}\sigma q} | \psi \rangle. \quad (2.19)$$

where $a_{\mathbf{r}'\sigma'q'}^+$ and $a_{\mathbf{r}\sigma q}$ are creation and annihilation operators which affect nucleon with $\sigma = \pm 1/2$ spin and q isospin from nucleon to hyperon at point r . The variation of the energy expectation value $E = \langle \Phi | \hat{H} | \Phi \rangle$, with respect to ρ and $\tilde{\rho}$ under the conservation of particle numbers, leads to HFB (or Bogoliubov-de Gennes) equations in coordinate space:

$$\begin{aligned} \int d^3\mathbf{r}' \sum_{\sigma'} \begin{pmatrix} h(\mathbf{r}\sigma, \mathbf{r}'\sigma') & \tilde{h}(\mathbf{r}\sigma, \mathbf{r}'\sigma') \\ \tilde{h}(\mathbf{r}\sigma, \mathbf{r}'\sigma') & -h(\mathbf{r}\sigma, \mathbf{r}'\sigma') \end{pmatrix} \begin{pmatrix} \psi_1(E, \mathbf{r}'\sigma') \\ \psi_2(E, \mathbf{r}'\sigma') \end{pmatrix} \\ = \begin{pmatrix} E + \lambda & 0 \\ 0 & E - \lambda \end{pmatrix} \begin{pmatrix} \psi_1(E, \mathbf{r}\sigma) \\ \psi_2(E, \mathbf{r}\sigma) \end{pmatrix}, \end{aligned} \quad (2.20)$$

where the mean field matrix elements ($h(\mathbf{r}\sigma, \mathbf{r}'\sigma')$ and $\tilde{h}(\mathbf{r}\sigma, \mathbf{r}'\sigma')$) are obtained by variation of the expectation value of the energy with respect to the particle and pairing densities:

$$h(\mathbf{r}\sigma, \mathbf{r}'\sigma') = \frac{\delta \epsilon(\rho_N, \rho_\Lambda, \tau_N, \tau_\Lambda)}{\delta \rho(\mathbf{r}\sigma, \mathbf{r}'\sigma')}, \quad (2.21)$$

$$\tilde{h}(\mathbf{r}\sigma, \mathbf{r}'\sigma') = \frac{\delta \epsilon(\rho_N, \rho_\Lambda, \tau_N, \tau_\Lambda)}{\delta \tilde{\rho}(\mathbf{r}\sigma, \mathbf{r}'\sigma')}. \quad (2.22)$$

In Eq. (2.20), the diagonal elements of the matrix in the integral correspond to the particle-hole (Hartree-Fock) field while the non-diagonal elements of the matrix correspond to particle-particle field which includes contributions of the pairing to the mean field of the hypernucleus. Recalling Eqs. (2.14, 2.15 and 2.16), one can rewrite the total energy functional for hypernucleus:

$$\begin{aligned} \epsilon(\rho_N, \rho_\Lambda, \tau_N, \tau_\Lambda) = \frac{\hbar}{2m_N^*} \tau_N + \frac{\hbar}{2m_\Lambda^*} \tau_\Lambda + \epsilon_{NN}^{nucl.}(\rho_N) \\ + \epsilon_{N\Lambda}^{nucl.}(\rho_N, \rho_\Lambda) + \epsilon_{\Lambda\Lambda}^{nucl.}(\rho_\Lambda), \end{aligned} \quad (2.23)$$

as it is shown in Eq. (2.21), taking derivatives of Eq. (2.23) leads to the mean field

operator for the particle-hole channel ($i = N, \Lambda$):

$$\int d^3\mathbf{r}' \sum_{\sigma'} h(\mathbf{r}\sigma, \mathbf{r}'\sigma') = \left[-\nabla \frac{\hbar^2}{2m_i^*(\mathbf{r})} \nabla + V_i(\mathbf{r}) - iW_i(\mathbf{r})(\nabla \times \sigma) \right], \quad (2.24)$$

where $W_i(\mathbf{r})$ is the spin-orbit term ($W_\Lambda(\mathbf{r}) = 0$ see Sec. 2.2 for details), $V_N(\mathbf{r})$ is the nucleon potential and $V_\Lambda(\mathbf{r})$ is Λ potential, respectively defined by:

$$V_N(\mathbf{r}) \equiv v_N^{Skyrme} + \frac{\partial \epsilon_{N\Lambda}}{\partial \rho_N} + \frac{\partial}{\partial \rho_N} \left[\frac{m_\Lambda}{m_\Lambda^*(\rho_N)} \right] \times \left[\frac{\tau_\Lambda}{2m_\Lambda} - \frac{3}{5} \frac{(3\pi^2)^{2/3} \hbar^2}{2m_\Lambda} \rho_\Lambda^{5/3} \right], \quad (2.25)$$

$$V_\Lambda(\mathbf{r}) \equiv \frac{\partial \epsilon_{N\Lambda}}{\partial \rho_\Lambda} + \frac{\partial \epsilon_{\Lambda\Lambda}}{\partial \rho_\Lambda} - \left[\frac{m_\Lambda}{m_\Lambda^*(\rho_N)} - 1 \right] \frac{(3\pi^2)^{2/3} \hbar^2}{2m_\Lambda} \rho_\Lambda^{2/3}. \quad (2.26)$$

2.3.3 The Particle-Particle Channel

For the particle-particle channel, due to scarce available information, especially for the Λ pairing channel, it is convenient to consider a volume type zero range pairing interaction in the $\Lambda\Lambda$ channel as,

$$V_{\Lambda_{pair}} = V_{\Lambda_0} \delta(\mathbf{r} - \mathbf{r}') \delta_{\sigma\sigma'}, \quad (2.27)$$

where V_{Λ_0} is the Λ pairing strength.

We now discuss the strength V_{Λ_0} of the Λ pairing interaction. At variance with the NN pairing interaction, there are not enough experimental data to set the $\Lambda\Lambda$ pairing interaction. We therefore choose to calibrate the $\Lambda\Lambda$ pairing interaction to calculations of Λ pairing gaps in uniform matter, see for instance the recent work in Ref. [85]. There are several predictions for the Λ pairing gap in uniform matter which have been employed in cooling models for neutron stars. These predictions are substantially different for several reasons: they were calibrated on either the old [71, 72] or the more recent [73, 75] value for the Nagara event [54]; they were considering non-relativistic [71, 72] or relativistic mean field [73, 75] approaches; as a consequence, they incorporate different density dependencies of the nucleon and Λ effective masses; they are based on various Λ interactions which are weakly constrained. As a result, qualitatively different predictions have been performed in nuclear matter: the influence of the nucleon density on the Λ pairing gap has been found opposite between non-relativistic approaches [71, 72] and relativistic ones [73]. Despite these differences, the predictions of the Λ pairing gap at saturation density

and for $k_{F_\Lambda} \approx 0.8 \text{ fm}^{-1}$ (corresponding to the average Λ density $\rho_{sat}/5$ in hypernuclei) are rather consistent across the different predictions and reach a maximum at about 0.5-0.8 MeV. For instance, under these conditions the Λ pairing gap is predicted to be about 0.5 MeV for $\rho_N = \rho_{sat}$ with HS-m2 parameters [73], and 0.5 (0.75) MeV for NL3 (TM1) parameters with ESC00 Λ force sets [75]. These values are also consistent with the extrapolations of earlier calculations [71, 72] in hypernuclear matter. Some interactions predict however lower values. In the following, we will therefore calibrate our $\Lambda\Lambda$ pairing interaction on hypernuclear matter predictions of Ref. [73], which represents an average prediction for the maximum possible Λ pairing gap.

In addition to the $\Lambda\Lambda$ pairing, let us mention the existence of a prediction suggesting a strong $N\Lambda$ pairing interaction in nuclear matter [74]. In finite nuclei, large $N\Lambda$ pairing gaps may however be quenched by shell effects, due to large single particle energy differences between the N and Λ states, or mismatch of the associated single particle wave functions. This will be discussed in more details in the next chapter of this thesis.

2.3.4 The Calculation of Nuclear Observables

Let us now discuss how to extract ground state properties for any hypernucleus with HFB framework. Considering Eq. (2.20), one can find energy eigenvalues and quasi-particle wave functions by solving two coupled differentials equation by using numerical iterations starting from a trial wave-function. However, we don't give the full detail about the numerical solution of HFB equations, we refer to Refs. [35, 36] for details. After the HFB iteration is converged, the particle and pairing densities can be expressed as

$$\rho(\mathbf{r}\sigma, \mathbf{r}'\sigma') = \sum_{E_n < E_{cut}} \psi_2(E_n, \mathbf{r}\sigma) \psi_2^*(E_n, \mathbf{r}'\sigma') \quad (2.28)$$

$$\tilde{\rho}(\mathbf{r}\sigma, \mathbf{r}'\sigma') = - \sum_{E_n < E_{cut}} \psi_2(E_n, \mathbf{r}\sigma) \psi_1^*(E_n, \mathbf{r}'\sigma'), \quad (2.29)$$

where E_{cut} is the cutoff energy which simulates the finite range of the interaction for mean fields. We used a 60 MeV cutoff energy and $15\hbar$ cutoff total angular momentum for quasi-particles, allowing for a large configuration space for all hypernuclei under study. The fact that the density matrix $\rho(\mathbf{r}\sigma, \mathbf{r}'\sigma')$ vanishes for $\mathbf{r} \rightarrow \infty$ or $\mathbf{r}' \rightarrow \infty$ has an important consequence for the canonical basis of the HFB problem formed by the eigenfunctions of the density matrix,

$$\int d^3\mathbf{r}' \sum_{\sigma'} \rho(\mathbf{r}\sigma, \mathbf{r}'\sigma') \phi_n(\mathbf{r}'\sigma') = v_n^2 \phi_n(\mathbf{r}\sigma), \quad (2.30)$$

where the v_n^2 is the occupation probability and $\phi_n(\mathbf{r}\sigma)$ are canonical states which also represents the wave function of nucleons. We now are ready to introduce pairing field $\tilde{h}(\mathbf{r}\sigma, \mathbf{r}'\sigma')$ by using Eq. (2.27) in the following way:

$$\tilde{h}(\mathbf{r}\sigma, \mathbf{r}'\sigma') = V_{\Lambda_{pair}} = V_{\Lambda_0} \delta(\mathbf{r} - \mathbf{r}') \delta_{\sigma\sigma'}. \quad (2.31)$$

Making the same approach as we did for Eq. (2.30), one can also generate u_n^2 (while satisfying $v_n^2 + u_n^2 = 1$ condition). As a result, quasi-particle wave functions ψ_1 and ψ_2 are now proportional to the canonical basis wave functions as,

$$\psi_1(E_n, \mathbf{r}\sigma) = u_n \phi_n(E_n, \mathbf{r}\sigma) \quad (2.32)$$

$$\psi_2(E_n, \mathbf{r}\sigma) = v_n \phi_n(E_n, \mathbf{r}\sigma), \quad (2.33)$$

which are also eigenfunctions of the particle-hole (Hartree-Fock) and particle-particle (pairing) field:

$$\int d^3\mathbf{r}' \sum_{\sigma'} h(\mathbf{r}\sigma, \mathbf{r}'\sigma') \phi_n(\mathbf{r}', \sigma') = (\epsilon_n - \lambda) \phi_n(\mathbf{r}, \sigma), \quad (2.34)$$

$$\int d^3\mathbf{r}' \sum_{\sigma'} \tilde{h}(\mathbf{r}\sigma, \mathbf{r}'\sigma') \phi_n(\mathbf{r}', \sigma') = \Delta_n \phi_n(\mathbf{r}, \sigma). \quad (2.35)$$

The eigenvalues $\epsilon_n - \lambda$ and Δ_n represent particle and pairing energies for each shell respectively where λ is acquired from the HFB iteration is already defined in Eq. (2.20). Besides, λ can also be interpreted as the Fermi energy of each channel when the HFB iteration is converged [36].

Finally we shall obtain the total energy of a hypernucleus and also density distribution of each nucleons and hyperons. The total energy of a hypernucleus E_{tot} can be calculated by using ϵ_n and Δ_n as

$$E_{tot} = \sum_{i=N, \Lambda} \sqrt{(\epsilon_i - \lambda)^2 + \Delta_i^2}. \quad (2.36)$$

The particle and pairing densities for an each shell is easily calculated by taking the trace of densities which are already defined in Eq. (2.28):

$$\rho_{N, \Lambda}(\mathbf{r}) = \text{Tr}[\rho_{N, \Lambda}(\mathbf{r}\sigma, \mathbf{r}'\sigma')], \quad (2.37)$$

$$\tilde{\rho}_{N, \Lambda}(\mathbf{r}) = \text{Tr}[\tilde{\rho}_{N, \Lambda}(\mathbf{r}\sigma, \mathbf{r}'\sigma')]. \quad (2.38)$$

In the following chapter, we show the calculated results of single particle spectrums, densities and binding energies for the selected hypernuclei.

2.4 Concluding Remarks

In the present chapter, we provided theoretical foundations for calculating ground state properties of Λ hypernuclei. On this purpose, we first described a density dependent interaction for NN, $N\Lambda$ and $\Lambda\Lambda$ channels. For NN channel, the SLy5 Skyrme functional is used, while for the $N\Lambda$ channel we use three functionals fitted from microscopic Brueckner-Hartree-Fock calculations: DF-NSC89, DF-NSC97a and DF-NSC97f. These functionals reproduce the sequence of single- Λ experimental binding energies from light to heavy hypernuclei. For the $\Lambda\Lambda$ channel, we used the empirical prescription EmpC which is calibrated on the experimental bond energy in ${}^6\text{He}_{\Lambda\Lambda}$ in which, we excluded the high density behavior of the $\Lambda\Lambda$ functional due to the lack of experimental information. In order to modelling pairing, we consider a zero range pairing force in the $\Lambda\Lambda$ channel, opening the possibility to calculate open- Λ shell nuclei. We also reviewed the current situation about Λ pairing in hypernuclear matter sector where, the Λ pairing gap varies between from 0.5 MeV to 0.75 MeV for the density $\rho_N = \rho_{sat}$ depending on different interactions. All things considered, we use all these interactions within the Hartree-Fock-Bogoliubov formalism to calculate ground state properties of Λ hypernuclei. In addition, our calculations are performed in spherical symmetry since deformation is not expected to increase greatly in the case of pairing correlations. Therefore, we shall consider hypernuclei with proton and neutron closed shells, e.g. ${}^{40}_{-\Lambda}\text{Ca}$, ${}^{132}_{-\Lambda}\text{Sn}$ and ${}^{208}_{-\Lambda}\text{Pb}$, since semi-magicity often guarantees that nuclei remain at, or close to, sphericity.

3.1 An Overview

In the present section, we show predictions for the Λ pairing gap and its impact on multi- Λ hypernuclei. In order to achieve this quest, we first discuss relative gaps between N and Λ single particle energies predicted by HF calculation without the pairing to assert our calculation without $N\Lambda$ pairing. We then employ Hartree-Fock-Bogoliubov (HFB) framework including pairing in the $\Lambda\Lambda$ channel to study binding energies and density profiles in multi- Λ hypernuclei.

3.2 Hunting Clues for the Λ Pairing: Results of the Hartree-Fock Approach

3.2.1 Λ Single Particle Spectrums for Multi- Λ Hypernuclei

Let us first discuss the hypernuclei of interest in this work, without $\Lambda\Lambda$ pairing interaction. On this purpose, we investigate closed shell hypernuclei such as $^{60}_{20\Lambda}\text{Ca}$, $^{172}_{40\Lambda}\text{Sn}$, $^{278}_{70\Lambda}\text{Pb}$ shown in Fig. 3.1. These nuclei are triply magic. Due to the absence of spin-orbit term, the shell structure of hyperon is also expected to be similar to that of the spherical harmonic oscillator, with magic numbers 2, 8, 20, 40, 70, etc, and the energy gaps are larger than in ordinary nuclei. While the central potential is also expected to modify the details of the single particle spectrum, the gross increase of the single particle gap compared to ordinary nuclei, is mostly due to the absence of spin-orbit interaction. New magic numbers in the Λ channel is clearly caused by absence of spin-orbit interaction, increasing the degeneracy of the states as well as the average energy gaps between single particle states. In order to check this statement, we have calculated the single particle spectrum for other Skyrme interactions, e.g. SGII [86], SAMI [87] and SIII [88], and we found the same gross conclusions.

The average single particle gaps between two neighboring orbitals can be estimated from Fig. 3.1, where the Λ spectrum is shown for $^{60}_{20\Lambda}\text{Ca}$, $^{172}_{40\Lambda}\text{Sn}$, and $^{278}_{70\Lambda}\text{Pb}$ hypernuclei

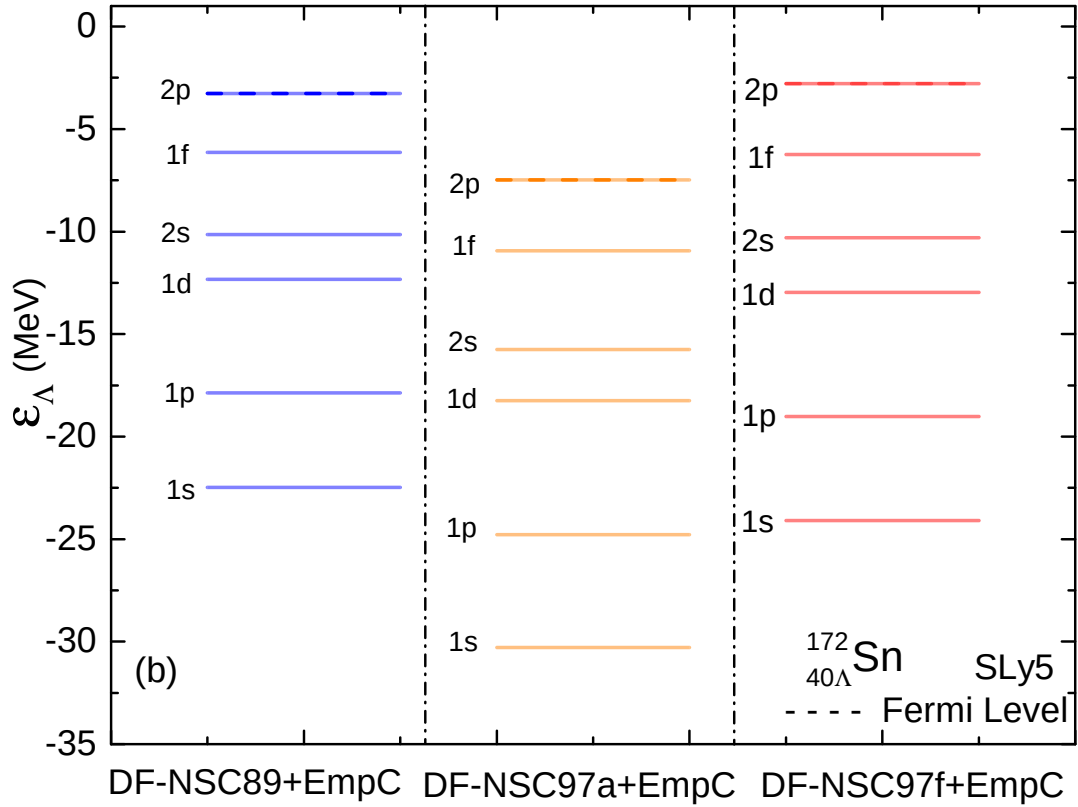
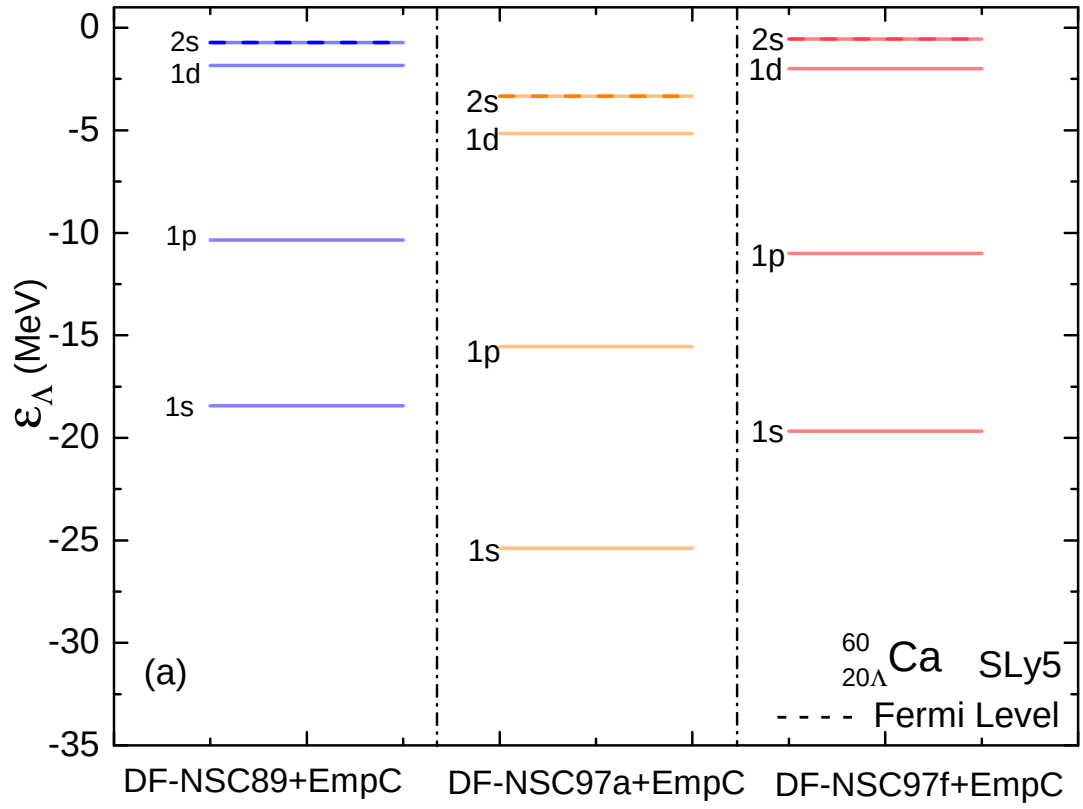


Figure 3.1 The Λ single particle spectrum $^{60}_{20\Lambda}\text{Ca}$ (a) and $^{172}_{40\Lambda}\text{Sn}$ (b) hypernuclei, calculated with the HF approach

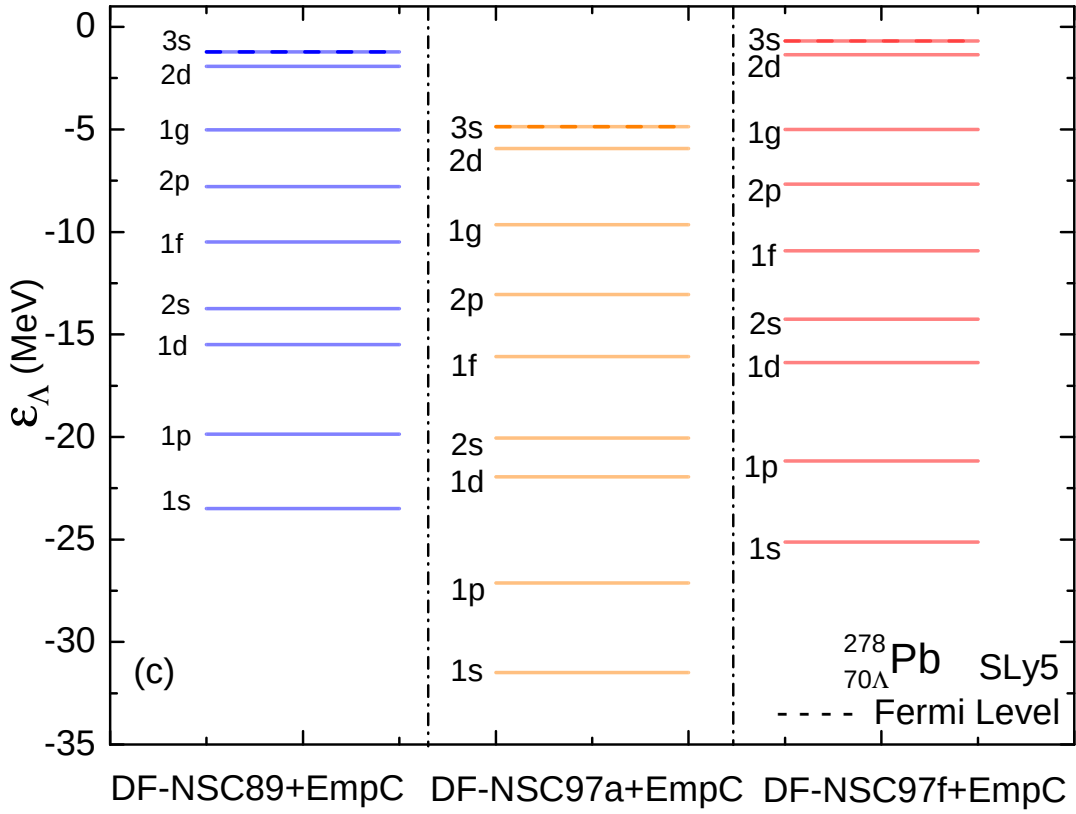


Figure 3.1 The Λ single particle spectrum of $^{278}_{70\Lambda}\text{Pb}$ (c) hypernucleus, calculated with the HF approach (Continued)

and for 3 different density functional in the Λ channel (the Skyrme interaction SLy5 is fixed in the nucleon channel): the average single-particle gap is found to be generally larger than 4 MeV, except for the gap between the 2s-1d and 3s-2d states, where it is between 1 and 3 MeV. These smaller energy gaps may be related to the pseudo-spin symmetry [89, 90], since the 2s-1d and 3s-2d states are pseudo-spin partners. The small energy gap between these states makes them good candidates for pairing correlations: These states could largely mix against pairing correlations when they are close to the Fermi level, represented in dashed lines in Fig. 3.1. For the selected nuclei in Fig. 3.1, the Fermi level is indeed close to either the 2s-1d or the 3s-2d states in the cases of Ca and Pb hypernuclei, respectively.

The energy spectra predicted by DF-NSC89+EmpC and DF-NSC97f+ EmpC are mainly identical, while the single particle states predicted by DF-NSC97a+ EmpC are systematically more bound, since the $N\Lambda$ potential is deeper for DF-NSC97a+ EmpC compared to the two others functionals [69, 83]. We give a more quantitative estimation of the single particle energy differences between the predictions of DF-NSC97a+EmpC and DF-NSC89+EmpC in Tab. 3.1. As expected, the larger the number of hyperons, the larger the differences. The larger the nucleon density, the

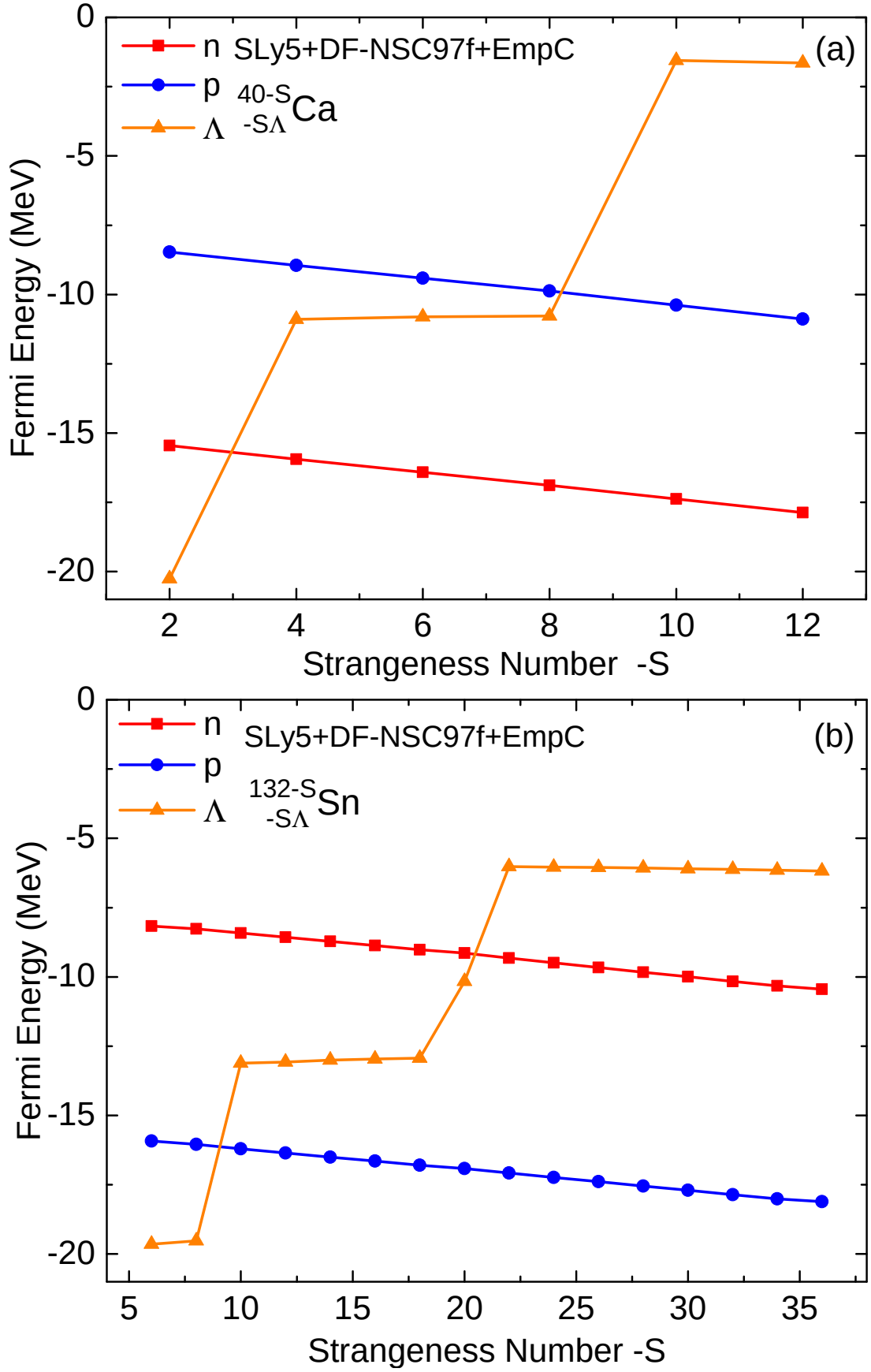


Figure 3.2 Evolution of proton, neutron and Λ Fermi energies function of strangeness number $-S$ for $^{40-S}_{-S\Lambda}\text{Ca}$ (a) and $^{132-S}_{-S\Lambda}\text{Sn}$ (b) hypernuclei with the HF approach

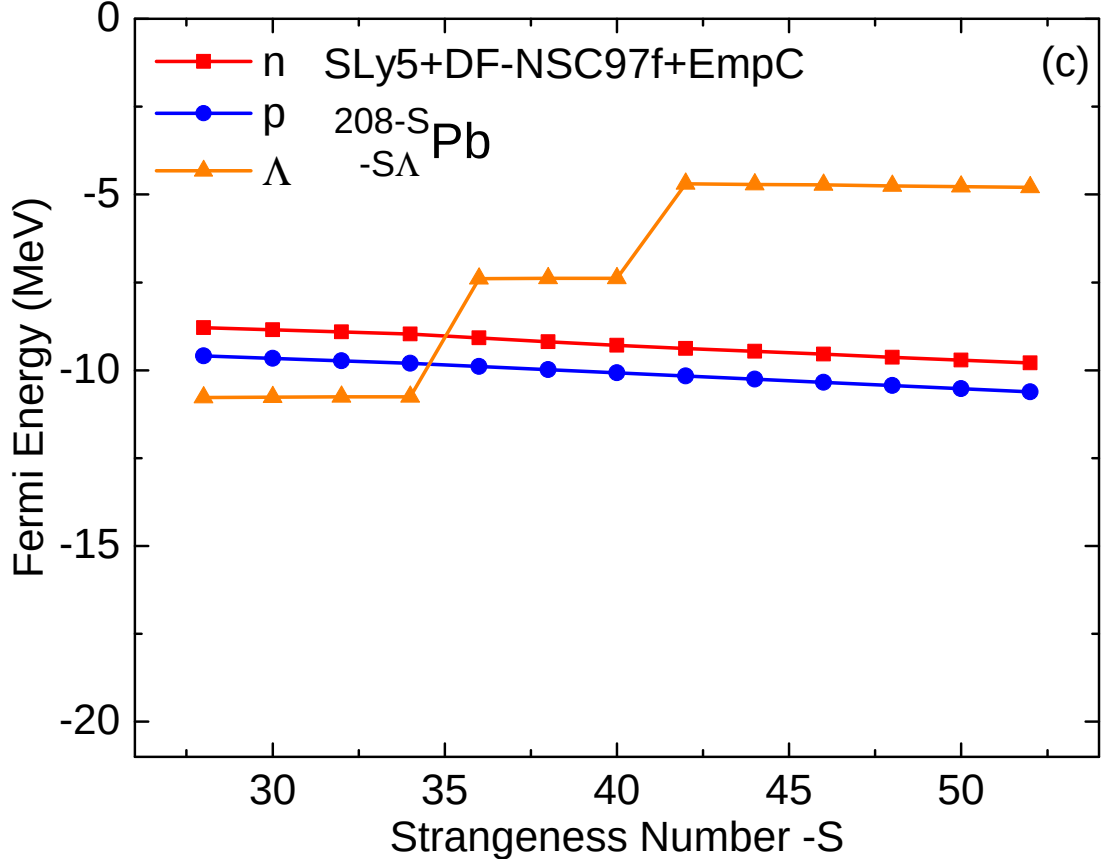


Figure 3.2 Evolution of proton, neutron and Λ Fermi energies function of strangeness number $-S$ for $^{208-S}_{-\Lambda}\text{Pb}$ (c) hypernuclei with the HF approach (Continued)

larger the differences as well, since the $N\Lambda$ potential strongly depends on nucleon density. Hence, the difference is larger for the deep single particle states than for the weakly bound ones.

3.2.2 Possibility of the $N\Lambda$ Pairing Channel

We now discuss the $N\Lambda$ and $\Lambda\Lambda$ pairing channels. These two pairing channels are expected to compete: a Lambda can be paired either to a nucleon or to another Lambda. Drawing an analogy with $T = 0$ and $T = 1$ pairing channels in ordinary nuclei, the pairing interaction between two different particles ($T = 0$) can occur under the condition of a good matching between their wave functions and also between their single particle energies. This is the main reason why $T = 0$ pairing is expected to appear mainly at (or close to) $N=Z$ nuclei [91, 92].

Let us first focus on the $N\Lambda$ pairing. A necessary condition for this pairing to occur is that the neutron or proton Fermi energy is close to the Λ one. The neutron, proton and Λ Fermi energies are displayed on Fig. 3.2 as function of the strangeness number $-S$ for the three representative nuclei: $^{40-S}_{-\Lambda}\text{Ca}$, $^{132-S}_{-\Lambda}\text{Sn}$ and $^{208-S}_{-\Lambda}\text{Pb}$. The intersections of

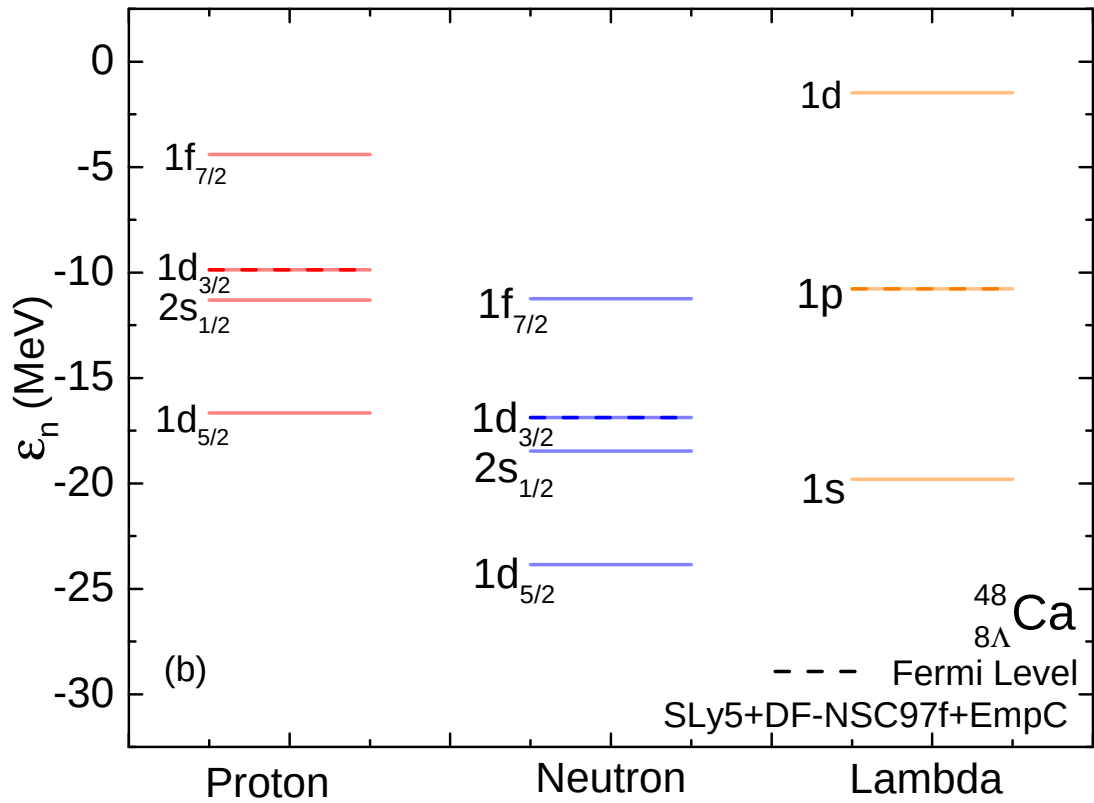
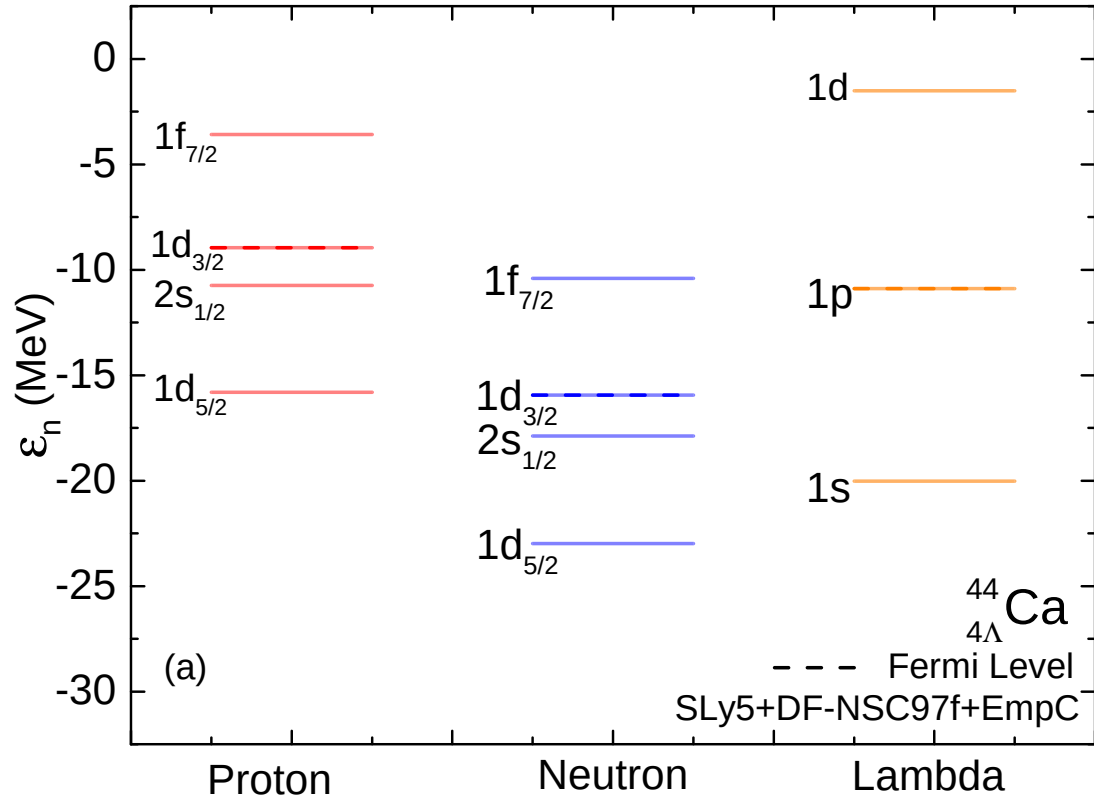


Figure 3.3 A complete single particle spectrum of $^{44}_{4\Lambda}\text{Ca}$ (a) and $^{48}_{8\Lambda}\text{Ca}$ (b) hypernuclei, calculated with the HF approach

Table 3.1 Energy difference of each shell between DF-NSC97a+EmpC and DF-NSC89+EmpC force sets. The detailed spectra are shown in Fig. 3.1

Shell	Energy Difference (MeV)		
	$^{60}_{20\Lambda}\text{Ca}$	$^{172}_{40\Lambda}\text{Sn}$	$^{278}_{70\Lambda}\text{Pb}$
1s	6.00	7.50	8.50
1p	6.00	6.87	7.10
1d	3.32	5.80	6.36
2s	2.59	5.57	6.42
1f	-	4.81	6.30
2p	-	4.20	5.20

nucleons and Λ Fermi energies occur at $-S = 4$ (neutrons) and 8 (protons) for $^{40-S}_{-S\Lambda}\text{Ca}$, $-S = 10 - 16$ (proton) and $20 - 32$ (neutrons) for $^{132-S}_{-S\Lambda}\text{Sn}$ and for $-S = 34 - 40$ for both neutrons and protons $^{208-S}_{-S\Lambda}\text{Pb}$ hypernuclei.

Let us now take typical examples of the nuclei which are located at these crossings. $^{44}_{4\Lambda}\text{Ca}$ and $^{48}_{8\Lambda}\text{Ca}$ single-particle levels are shown in Fig. 3.3 and the ones of $^{244}_{36\Lambda}\text{Pb}$ is shown in Fig. 3.4. The Λ Fermi level is mainly the 1p state in $^{44}_{4\Lambda}\text{Ca}$ and $^{48}_{8\Lambda}\text{Ca}$, and there are no p states in the neutron and proton spectrum around the Fermi energy. The conditions for NA pairing are therefore not satisfied for Ca isotopes.

A similar analysis can be made for the Sn isotopes. We also calculated $^{142}_{10\Lambda}\text{Sn}$, $^{152}_{20\Lambda}\text{Sn}$ and $^{156}_{24\Lambda}\text{Sn}$ hypernuclei for which the proton or neutron and the Λ levels cross. The last occupied Λ states is 1d for $^{142}_{10\Lambda}\text{Sn}$ (resp. 2s for $^{152}_{20\Lambda}\text{Sn}$ and 1f for $^{156}_{24\Lambda}\text{Sn}$) while the corresponding proton (res. neutron) state is $1g_{9/2}$ (resp. $1h_{11/2}$). Since the quantum orbital numbers does not coincide between the nucleons and the Λ states in the cases where their respective Fermi energies cross, the NA pairing is not favored for these Ca and Sn nuclei.

The case of $^{208-S}_{-S\Lambda}\text{Pb}$ hypernuclei is different. Fig. 3.4 displays the single particle spectrum for $^{244}_{36\Lambda}\text{Pb}$ hypernucleus, since the crossing of the nucleon (neutrons and protons) and Λ Fermi levels occurs at around $S = -36$ (Fig. 3.3). Fig. 3.4 shows that the last filled orbits are $3s_{1/2}$ for proton, $3p_{1/2}$ for neutron and 2p for Λ . Since Pb is magic in proton, only neutrons and Λ may be paired. We therefore predict that n Λ pairing may occur for $^{208-S}_{-S\Lambda}\text{Pb}$ hypernuclei and for Λ between $S = -34$ and $S = -40$. For lower or higher values of S , the mismatching of the single particles orbitals does not favor n Λ pairing. Since the level density increases with increasing masses, it is expected the general trend that NA pairing may occur more frequently as A increases.

$^{208-S}_{-S\Lambda}\text{Pb}$ is a typical case representing heavy hypernuclei. Since the Ξ -instability is

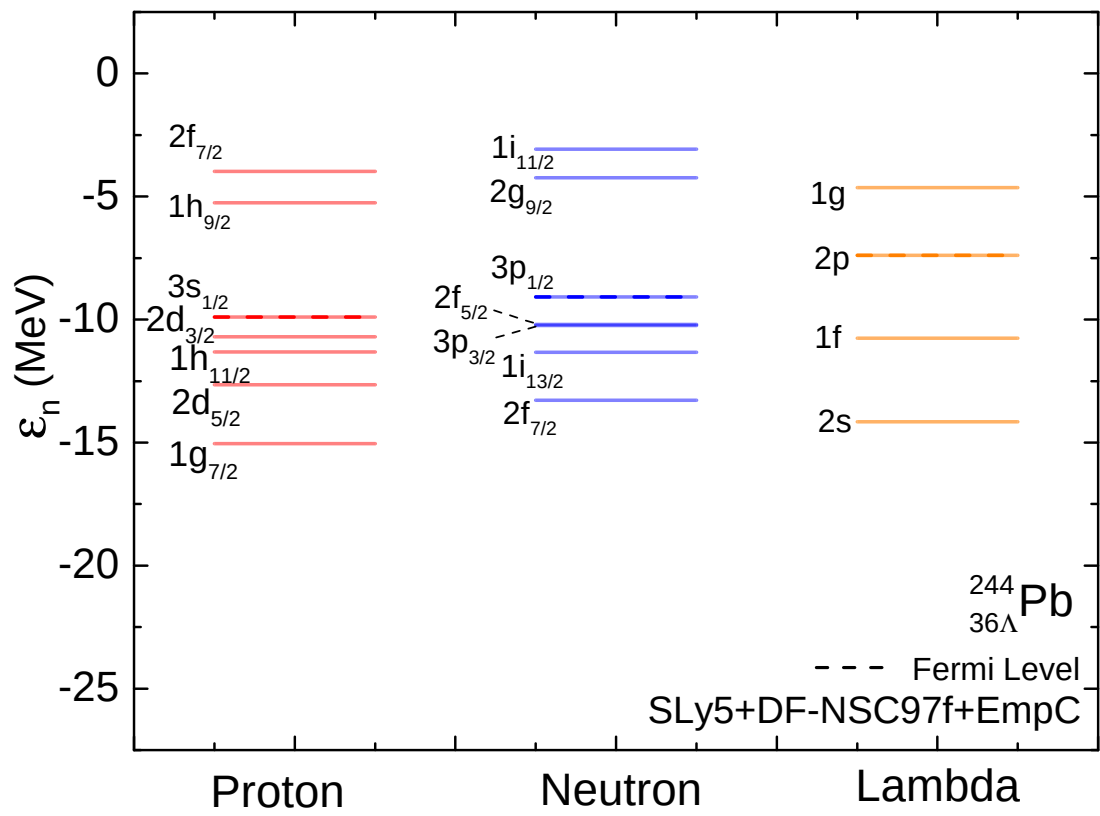


Figure 3.4 A complete single particle spectrum of $^{244}_{36\Lambda}\text{Pb}$ hypernucleus with the HF approach

expected to occur around $-S = 70$ [69], we can infer that pairing may occur for about 10% of $^{208-S}_{-S\Lambda}\text{Pb}$ isotopes. This number may be considered as the maximum percentage of heavy hypernuclei where $\Lambda\Lambda$ pairing may occur. The amount of hypernuclei where $\Lambda\Lambda$ pairing is possible is therefore predicted to be small. In the following, we will avoid the cases where it may occur.

3.3 Investigating Impacts of the $\Lambda\Lambda$ Pairing on Hypernuclei: Results of the Hartree-Fock-Bogoliubov Approach

3.3.1 A Fitting Protocol for the $\Lambda\Lambda$ Pairing Strength

We now focus on the $\Lambda\Lambda$ pairing and consider the cases of semi-magical hypernuclei, such as $^{40-S}_{-S\Lambda}\text{Ca}$, $^{132-S}_{-S\Lambda}\text{Sn}$, and $^{208-S}_{-S\Lambda}\text{Pb}$. It should be noted that these nuclei are magic in both proton and neutron numbers, which helps most of these hypernuclei to resist against deformation, as in the case of normal hypernuclei. They have however an open shell in the Λ channels.

The Λ pairing strengths, mean gaps and averaged mean gaps of isotopic chains are displayed in Tab. 3.2. The fitting procedure for the $\Lambda\Lambda$ pairing is the following: we first remind that the $\Lambda\Lambda$ mean-field interaction is calibrated to the $\Lambda\Lambda$ bond energy in ^6He (Nagara event; see Sec. 2.3.1 for details). We then consider open-shell nuclei and calibrate the average Λ -pairing gap to its expectation from uniform matter calculations. Densities are averaged from $r=0.2$ fm to 10 fm for each hypernucleus and each force set using HF results for each isotopes. Fermi momentums corresponding to these densities are calculated as $k_{F\Lambda} = (\frac{3\pi^2}{2}\rho_\Lambda)^{1/3}$. The density profile of hypernuclear matter calculations [73] which has corresponding Fermi momentum and density fraction allows to extract $\Lambda\Lambda$ pairing gap for each hypernuclei for each force sets. In order to find adequate Λ pairing strength ($V_{\Lambda_{pair}}$ in Eq. (2.27)), starting from -50 MeV fm³ to -300 MeV fm³, we iterated and determined mean gap values for each hypernuclei chain in HFB calculation. For each iteration, the mean gap values are averaged over the isotopic chain until similar values of pairing gaps of hypernuclear matter calculation are obtained. Namely for the $^{40-S}_{-S\Lambda}\text{Ca}$ isotopic chain, the average mean gap was calculated by summing each mean gap of hypernucleus starting from $-S=6$ to $-S=20$ and dividing by the total isotope number. Similarly for the $^{132-S}_{-S\Lambda}\text{Sn}$ ($^{208-S}_{-S\Lambda}\text{Pb}$) isotopes, the average man gap determined between $-S=18$ ($-S=58$) to $-S=40$ ($-S=70$) range. The average mean gaps for each isotopes with each force set is given in Tab. 3.2. A typical 0.5 MeV gap is obtained in all nuclei, leading to a pairing effect independent of the number of Λ involved.

Table 3.2 Pairing strength, Λ density and calculated, averaged mean gap and hypernuclear pairing gap for each force sets

Force Set	Pairing Strength (MeV fm ³)	ρ_{av} (fm ⁻³)	Hypernucleus	Calculated		Average		Pairing Gap in	
				Mean Gap (MeV)	Mean Gap (MeV)	Mean Gap (MeV)	Mean Gap (MeV)	uniform matter [73] (MeV)	uniform matter [73] (MeV)
DF-NSC89+EmpC	-139	0.0264	⁴⁶ Ca _{6Λ}	0.82	0.82	0.59	0.59	0.42	0.42
DF-NSC97a+EmpC	-148	0.0349	⁴⁶ Ca _{6Λ}	1.04	1.04	0.50	0.50	0.44	0.44
DF-NSC97f+EmpC	-180	0.0241	⁴⁶ Ca _{6Λ}	0.98	0.98	0.49	0.49	0.43	0.43
DF-NSC89+EmpC	-158	0.0421	¹⁶⁰ Sn _{28Λ}	0.84	0.84	0.46	0.46	0.43	0.43
DF-NSC97a+EmpC	-145	0.0499	¹⁶⁰ Sn _{28Λ}	0.82	0.82	0.45	0.45	0.50	0.50
DF-NSC97f+EmpC	-180	0.0382	¹⁶⁰ Sn _{28Λ}	0.82	0.82	0.46	0.46	0.45	0.45
DF-NSC89+EmpC	-184	0.0400	²⁷² Pb _{64Λ}	0.69	0.69	0.44	0.44	0.47	0.47
DF-NSC97a+EmpC	-180	0.0320	²⁷² Pb _{64Λ}	0.76	0.76	0.46	0.46	0.45	0.45
DF-NSC97f+EmpC	-220	0.0270	²⁷² Pb _{64Λ}	0.71	0.71	0.44	0.44	0.40	0.40

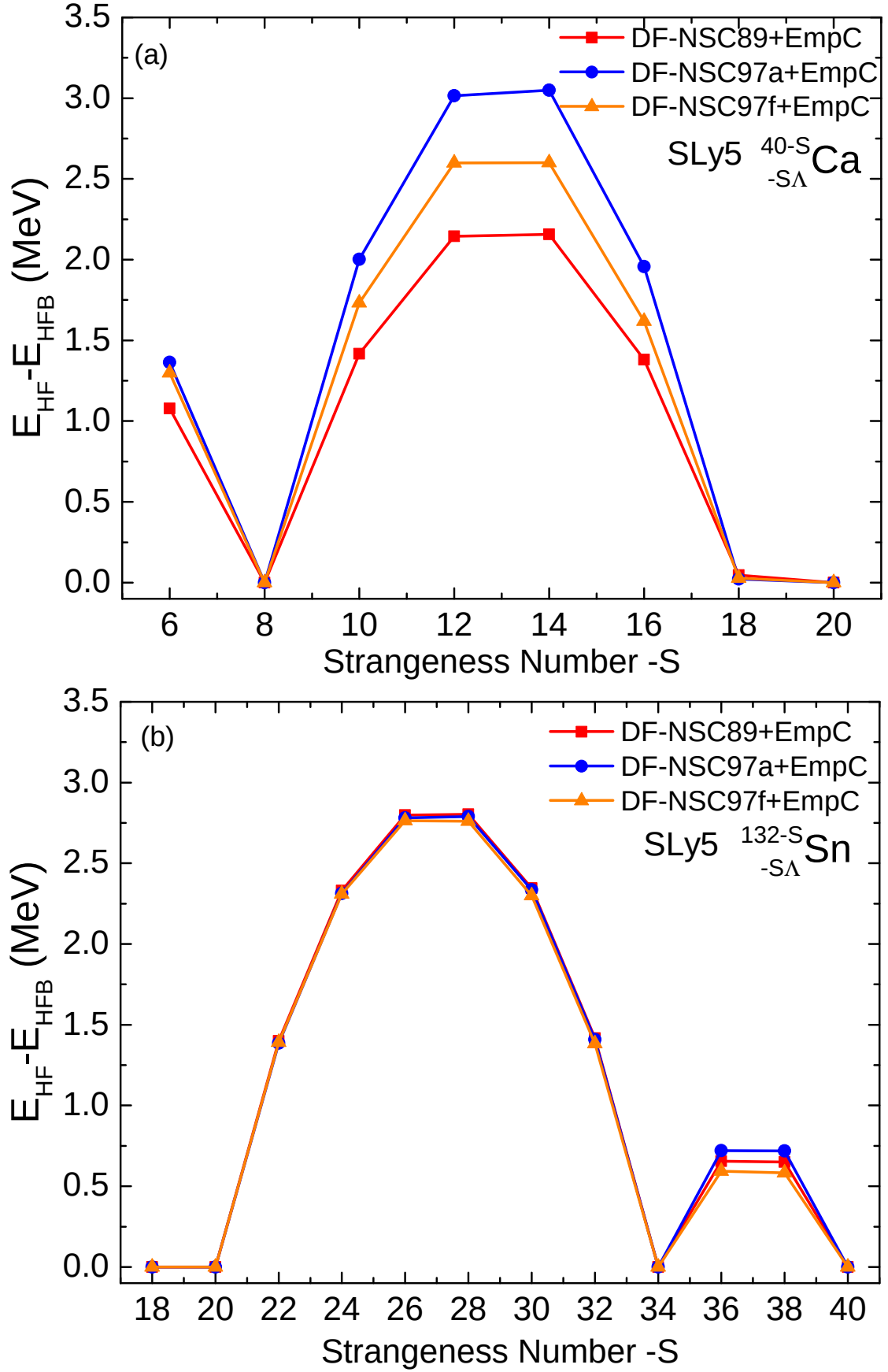


Figure 3.5 Difference of binding energies between HF and HFB for $^{40-S}_{-S\Lambda}\text{Ca}$ (a) and $^{132-S}_{-S\Lambda}\text{Sn}$ (b) hypernuclei with DF-NSC89+EmpC, DF-NSC97a+EmpC and DF-NSC97f+EmpC force sets

3.3.2 The $\Lambda\Lambda$ Pairing and Nuclear Binding Energies

The effect of $\Lambda\Lambda$ pairing on the binding energy can be estimated from the condensation energy, defined as $E_{cond} = E_{HF} - E_{HFB}$. The condensation energy measures the impact of the pairing effect on the binding energy. Fig. 3.5 displays the condensation energy for a set of $^{40-S}_{-\Lambda}\text{Ca}$ and $^{132-S}_{-\Lambda}\text{Sn}$ semi-magical hypernuclei. As in the case of normal nuclei, the condensation energy evolves as arches, with zero values at closed shells and maximum values for middle-open shells. The condensation energy can reach about 3 MeV in mid-open shell hypernuclei for Ca and Sn isotopes. Since the $\Lambda\Lambda$ pairing interaction considered here is calibrated on the maximum prediction for the Λ gap in uniform matter with respect to Λ force sets, the condensation energy represents the estimation of the maximum value for the condensation energy generated by $\Lambda\Lambda$ pairing interaction. Besides, the Λ numbers at which condensation energy is zero signs the occurrence of the shell closure. It is therefore not surprising to recover the magical numbers 8, 20, 40, as we previously discussed. Strong sub-shell closure occurs for $\Lambda=34$ in Sn isotopes corresponding the filling of the 1f state.

Investigating the effect of $\Lambda\Lambda$ pairing on the single particle energies, it turns out to be weak: states around the Fermi level are shifted by about 100-200 keV at maximum. The impact of Λ pairing on single particle energies remains therefore rather small.

3.3.3 The $\Lambda\Lambda$ Pairing and Nuclear Densities

We now discuss the effect of $\Lambda\Lambda$ pairing on both normal and pairing densities. Fig. 3.6 shows normal density profiles for $^{40-S}_{-\Lambda}\text{Ca}$, $^{132-S}_{-\Lambda}\text{Sn}$ and $^{208-S}_{-\Lambda}\text{Pb}$ series of hypernuclei. For the $^{40-S}_{-\Lambda}\text{Ca}$ series we consider cases where the $N\Lambda$ pairing is not expected to occur. As mentioned above, the 1d and 2s states are almost degenerate, and can largely mix due to pairing correlations. Namely, before the 1d orbital is completely filled, Λ hyperons start to fill the 2s state due to the pairing interaction, resulting in a small increase at the centre of the hypernucleus which corresponds the effect of the s state. Similar results can be seen on the density profile of $^{208-S}_{-\Lambda}\text{Pb}$ hypernucleus: Before the 2d state is completely filled, Λ hyperons start to fill the 3s state due to the pairing interaction resulting from the almost degeneracy of the 2d and 3s Λ -states. In the case of $^{132-S}_{-\Lambda}\text{Sn}$, there is no major difference on density profiles: because of the large gap between 1f and 2p states, the Λ pairing changes only the total energy of the $^{132-S}_{-\Lambda}\text{Sn}$ isotopic chain in $-S=24$ to $-S=30$ zone but does not impact the occupation numbers of 1f and 2p orbitals.

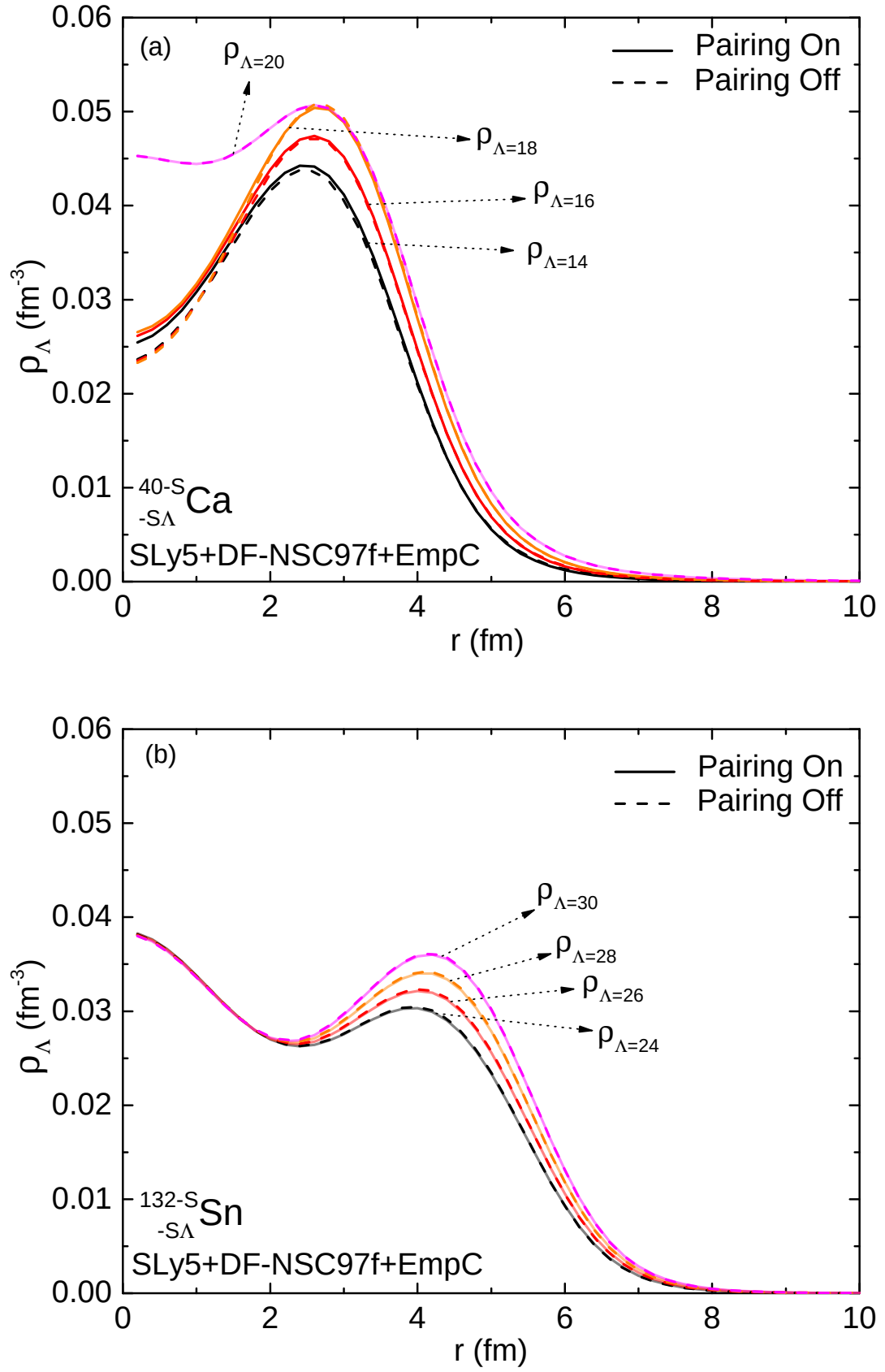


Figure 3.6 Normal density profiles with on/off pairing for $^{40-S}_{-S\Lambda}\text{Ca}$ (a) and $^{132-S}_{-S\Lambda}\text{Sn}$ (b) hypernuclei, calculated with the HFB approach

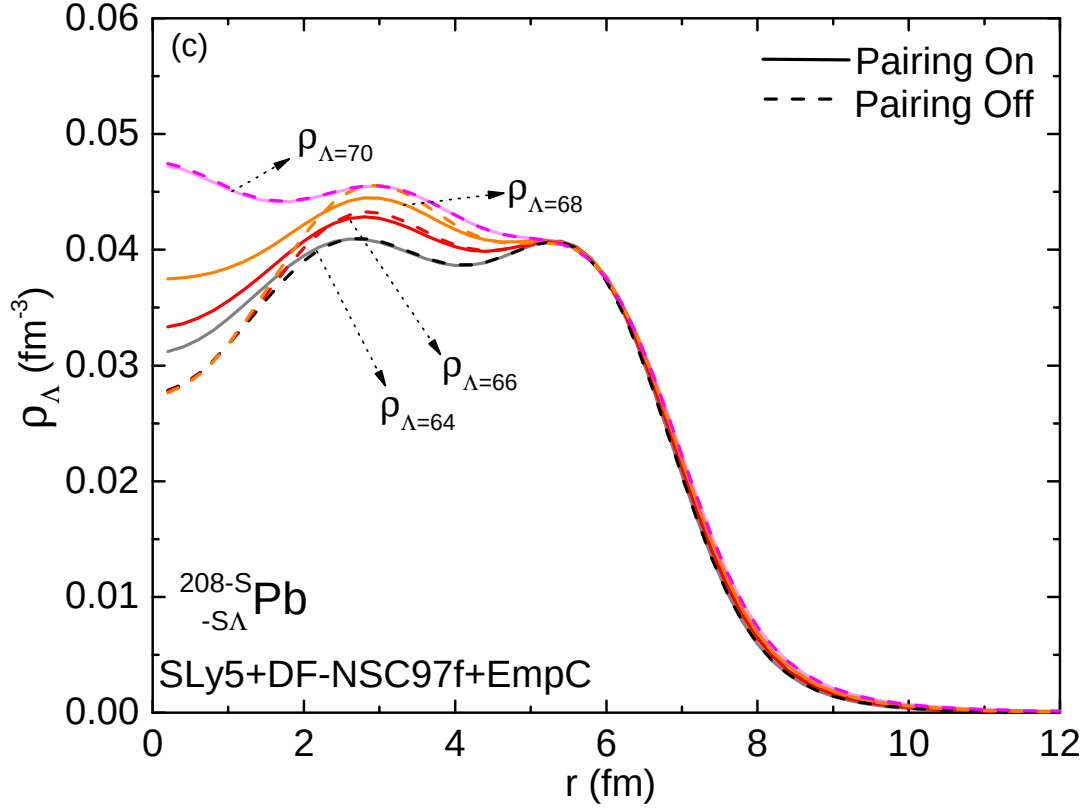


Figure 3.6 Normal density profiles with on/off pairing for $^{208-S}_{-S\Lambda}\text{Pb}$ (c) hypernuclei, calculated with the HFB approach

Fig. 3.7 displays the Λ pairing density for $^{40-S}_{-S\Lambda}\text{Ca}$, $^{132-S}_{-S\Lambda}\text{Sn}$ and $^{208-S}_{-S\Lambda}\text{Pb}$. As mentioned above, pairing interaction effects result in the partial occupation of Λ hyperons in the s and d states. The pairing density of $^{54}_{14\Lambda}\text{Ca}$ hypernucleus peaks at 3 fm due to half-filled 1d orbital. As strangeness number increases, hyperons start to fill the 2s state and the contribution of the 1d state decreases. For $-S=18$, Λ hyperons start to largely fill the 2s state before the 1d state is completely full, resulting in a pairing density having non-negligible contributions of both s and d states. Similar result can be seen for the pairing density of $^{208-S}_{-S\Lambda}\text{Pb}$ hypernuclei which has 2d-3s coupling. At $-S=64$, pairing densities are mainly built from the 2d state but as the strangeness number increases, the pairing of 2d orbital decreases while pairing density at 3s state increases. However for $^{132-S}_{-S\Lambda}\text{Sn}$ hypernuclei, the situation is different. Due to the large energy gap between 2s and 1f states, the pairing interaction does not change the occupation of these states. For this reason, the pairing density is only built from the 1f orbital and its magnitude increases when the occupation of the 1f orbital increases until it is half-filled. When the 1f state is more than half-filled, the magnitude of the pairing density starts to decrease. The spatial distribution of pairing effect in hypernuclei is therefore expected to exhibit strong variations from one hypernucleus to another, because of the weak spin-orbit

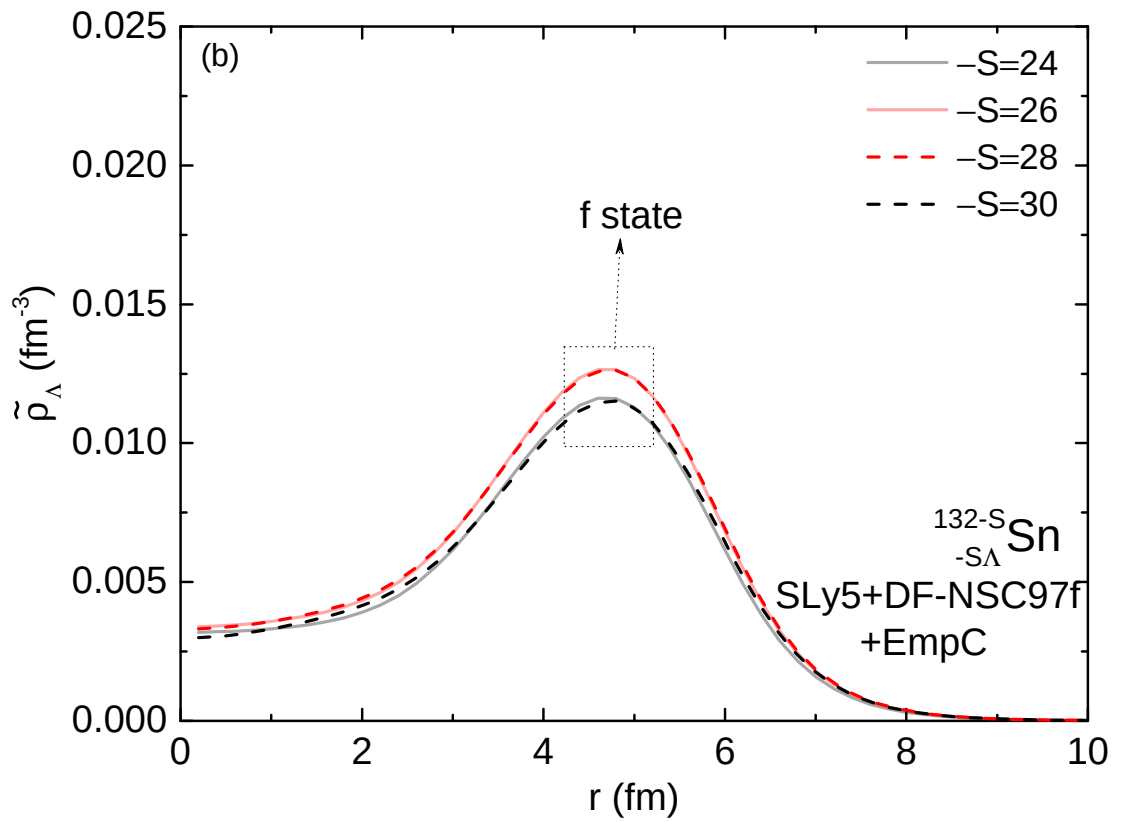
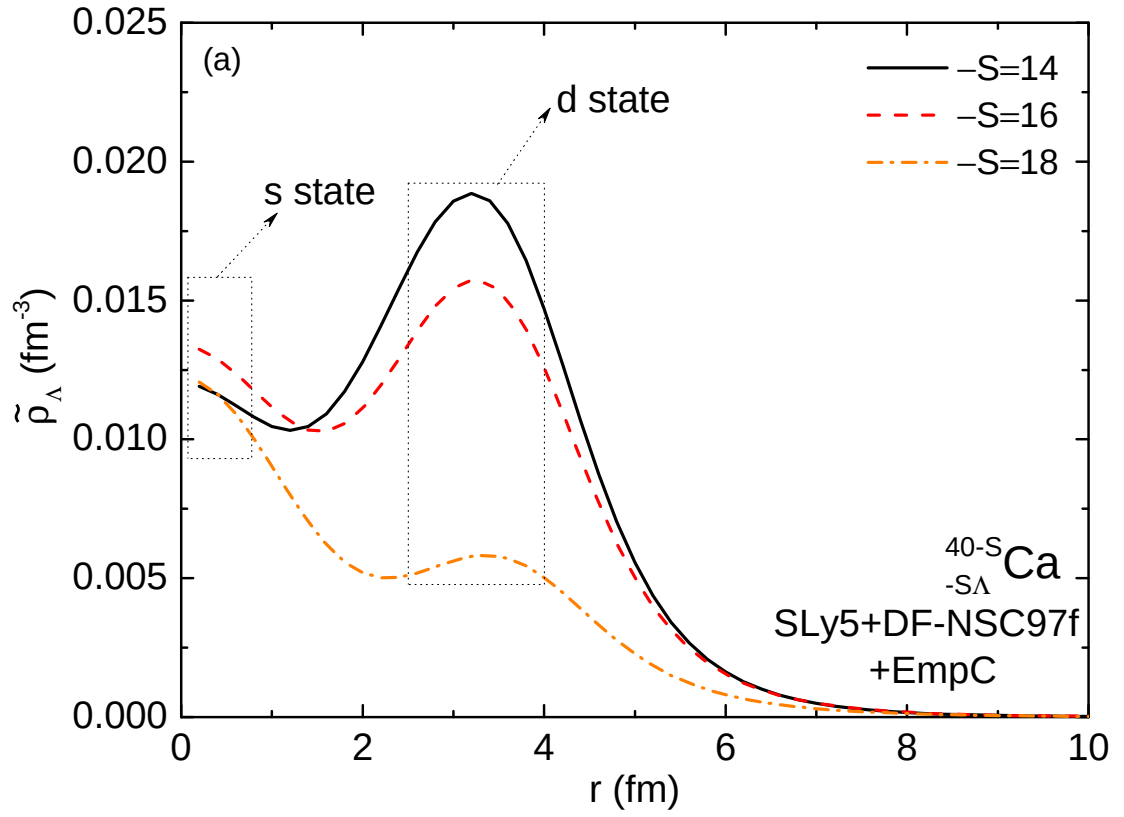


Figure 3.7 Λ pairing densities for $^{40-S}_{-S\Lambda}\text{Ca}$ (a) and $^{132-S}_{-S\Lambda}\text{Sn}$ (b) hypernuclei, calculated with the HFB approach

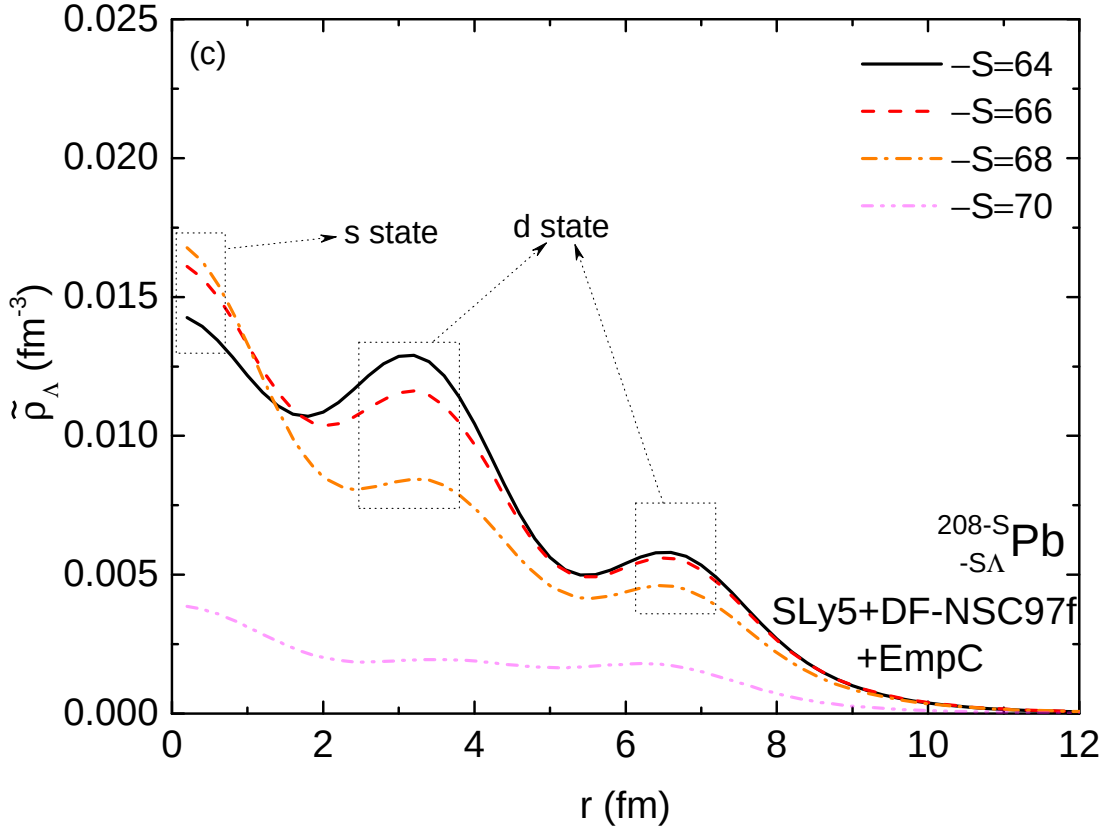


Figure 3.7 Λ pairing densities for $^{208-S}_{-S\Lambda}\text{Pb}$ (c) hypernuclei, calculated with the HFB approach

effect, giving rise to well separated sets of states.

3.4 Concluding Remarks

In the present chapter, we first investigated the shell structure of $^{40-S}_{-S\Lambda}\text{Ca}$, $^{132-S}_{-S\Lambda}\text{Sn}$ and $^{208-S}_{-S\Lambda}\text{Pb}$ hypernuclear chain with using HF approach to discuss the $\text{N}\Lambda$ pairing. It is seen that since the energy difference between the N and Λ Fermi levels are usually large (more than 5 MeV) in the considered hypernuclei, the $\text{N}\Lambda$ pairing is quenched in most of the cases. An exception however is $^{208-S}_{-S\Lambda}\text{Pb}$ hypernuclear chain. There is a match between $S = -34$ and $S = -40$ for neutron and Λ Fermi energies which means that neutron- Λ pairing may occur for the related interval. We can infer that pairing may occur for about 10% of $^{208-S}_{-S\Lambda}\text{Pb}$ isotopes. Besides, the expected general trend is that $\text{N}\Lambda$ pairing may occur more frequently as A increases because the level density increases with increasing masses. However, since the Ξ -instability could limit the maximum number of Λ around $-S = 70$, the amount of hypernuclei where $\text{N}\Lambda$ pairing is possible, is therefore predicted to be small.

We then used the HFB approach on same hypernuclei. A $\Lambda\Lambda$ pairing interaction

is therefore introduced, which magnitude is calibrated to be consistent with the maximum BCS predictions for the Λ pairing gap in hypernuclear matter. The impact of $\Lambda\Lambda$ pairing on the binding energies, density profiles and single particle energies have been analyzed for $^{40-S}_{-S\Lambda}\text{Ca}$, $^{132-S}_{-S\Lambda}\text{Sn}$ and $^{208-S}_{-S\Lambda}\text{Pb}$ chains. We have shown that the effects of the $\Lambda\Lambda$ pairing depends on hypernuclei. At maximum, the condensation energy in these chains is about 3 MeV. Density profiles reflect the occurrence of almost degenerate states in the Λ single particle spectrum, such as for instance the almost degeneracy between the 1d and 2s states in $^{40-S}_{-S\Lambda}\text{Ca}$ hypernuclei and 2d and 3s almost-degeneracy in $^{276}_{68\Lambda}\text{Pb}$. The effects of the Λ pairing also depend on the $N\Lambda$ and $\Lambda\Lambda$ force sets, but we found only a small overall impact. Generally, we found that $\Lambda\Lambda$ pairing could be active if the energy gap between orbitals is smaller than 3 MeV. Under this condition, Λ pairing could impact densities and binding energies. Since only a weak spin-orbit interaction is expected in the Λ channel, Λ states are highly degenerated and usually distant by more than 3 MeV in energy. In conclusion, the present microscopic approach shows that the Λ -related pairing effect can usually be neglected in most of hypernuclei, except for hypernuclei which have a single particle gap lower than 3 MeV around the Fermi level.

4.1 Prologue: A Dying Star

An ordinary star is at a delicate balance between the gravitation and the pressure caused by thermonuclear reactions. Since the fusion of light nuclei into ever increasing heavier elements terminates abruptly with the synthesis of the iron-group elements (mostly ^{56}Fe) that are characterized by having the largest binding energy per nucleon, the ^{56}Fe accretion continues at the core of the star as long as it has fuel to support thermonuclear reaction. Once, ^{56}Fe accretion at the core reaches the Chandrasekhar limit ($1.44M_{\odot}$) or its fuel to ignite thermonuclear reaction is finished, the gravity starts to win and the collapse begins [93, 94]. If the mass of star is greater than $9M_{\odot}$, the ^{56}Fe scenario happens and during the collapse, the star goes into the supernovae phase [94]. However, if it has a smaller mass than $9M_{\odot}$, the out-of-fuel scenario happens and the star will turn to a red giant [93]. In both cases, the thermonuclear reaction inside the core is slowed and eventually stopped. Since the thermonuclear reaction is the only source of pressure to counterbalance the gravity, the gravity starts to compress the interior of the star. First solid state forces try to stop gravity, however it is not enough: electrons start to be compressed by gravity while they resist placing at same quantum state due to their fermionic nature. This is called electron degeneracy pressure [3].

If the electron degeneracy pressure is large enough to stop the gravity, the dying star core becomes a white dwarf [3, 7, 8]. This is a typical end for red giants. However, if the dying star is a supernovae, the degeneracy pressure of the electrons is not large enough to stop the gravity. Therefore, the electrons start to get close from each other and their wavelength become of the order of fm, with an increased Fermi momentum allowing for appearing electrons near or inside of the nucleus at high pressure. This starts a massive electron capture process:



Because of electron capture, there is also a β decay:

$$n \longrightarrow p + e^- + \bar{\nu} \quad (4.2)$$

Consequently, ${}^{56}_{26}\text{Fe}$ turns to ${}^{62}_{28}\text{Ni}$ by electron capture process and subsequent β decay (for instance, 62 ${}^{56}_{26}\text{Fe}$ nuclei create 56 ${}^{62}_{28}\text{Ni}$ nuclei with the help of highly energetic degenerate electrons due to the gravity) then to a heavier neutron rich nucleus ${}^{122}_{39}\text{Y}$ and it continues to heavier nuclei with an increased neutron fraction [8]:

$${}^{56}_{26}\text{Fe} \xrightarrow{\text{Gravity}} {}^{62}_{28}\text{Ni} \xrightarrow{\text{Gravity}} {}^{122}_{39}\text{Y} \xrightarrow{\text{Gravity}} \dots \xrightarrow{\text{Gravity}} \text{Nuclear Matter} \quad (4.3)$$

Since the driven force is the work done by the compression due to the gravity, it continues until the gravity is counterbalanced: nuclear matter could stop the collapse due to its incompressibility. If then, accelerated particles during collapse will bounce from the nuclear matter core and create a shockwave which is called supernovae explosion [94]. Consequently, the matter outside of the core is ejected to the interstellar medium. The remaining neutron rich, hot and dense nuclear matter in the core forms an object named Neutron Star.

4.2 Neutron Stars: General Properties

Neutron stars (NS) are one of the most interesting objects in universe. They have a typical mass $M = 1 - 2M_{\odot}$, where $M_{\odot} = 2 \times 10^{33}$ g is the solar mass, and a radii $R = 10 - 14$ km [2, 3, 7, 8]. Their place on stellar evolution between white dwarfs and black holes, makes them a very charming object to understand the exotic phases of matter. While experiments in finite nuclei probe densities around saturation density of nuclear matter ($n_{\text{sat}} \approx 0.16\text{fm}^{-3}$, $\rho_{\text{sat}} \approx 2.7 \times 10^{14}$ g/cm³) and heavy-ion collisions explore a wider domain of densities with small isospin asymmetries, NSs are the solely system to explore the equilibrium properties of dense matter at densities well above saturation density, and with large isospin asymmetries (Note that instead of Ch. 2 and Ch. 3, we used n for the particle density and ρ for the matter density.) [2].

From the astrophysical side, the observation of NSs allows to set limits on the maximum mass which also affects the maximum density of stable baryonic matter. The maximum mass of neutron stars, which is yet not well-determined, fixes the mass boundary between NSs and black holes, which give clues on the understanding of supernova core-collapse mechanism [94] as well as of the fate of NS mergers as kilonovae [95]. The observed masses vary from $1.174(4)M_{\odot}$ [6, 70] to about $2M_{\odot}$ [6, 23] and the centroid value is $1.4 M_{\odot}$ [6]. The well established upper mass limits are: $1.908(16)M_{\odot}$ for PSR J1614-2230 [24] and $2.01(4)M_{\odot}$ for PSR

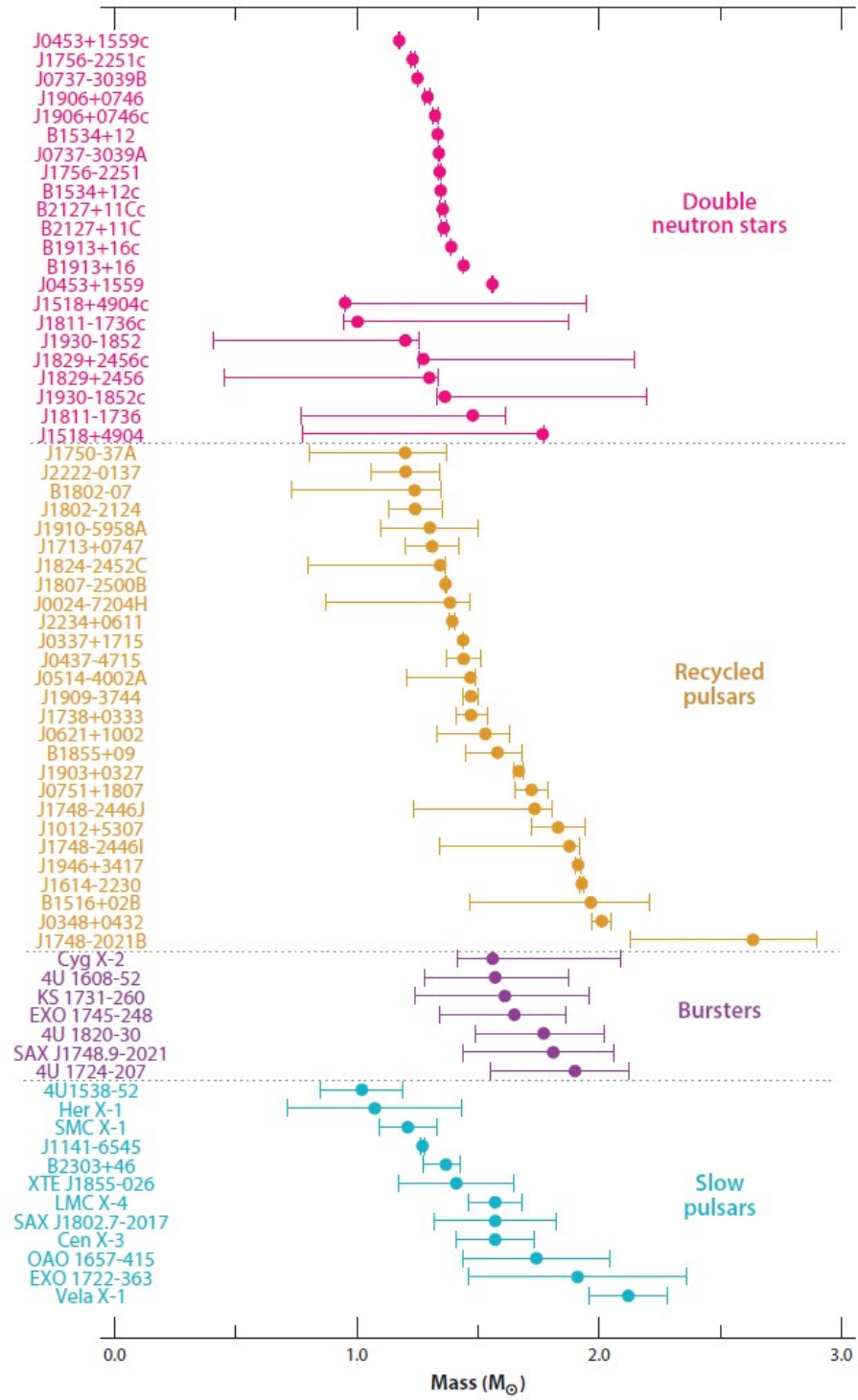


Figure 4.1 Observed NS masses from Ref. [6]

J0348+0432 [23]. Nowadays, widely accepted observed maximum mass of NS is ($M_{max}^{obs} = 2.01(4)M_{\odot}$) [23]. Recently, two new observations have raised up the upper limit to $M_{max} = 2.14_{-0.09}^{+0.10}M_{\odot}$ from Shapiro delay (which is an observable when a stellar object passes behind its companion during the orbital motion, creates a small delay in pulse arrival times induced by the curvature of space-time in the vicinity of the companion star) associated to the MSP J0740+6620 [96] and $M_{max} = 2.27_{-0.15}^{+0.17}M_{\odot}$ from magnesium lines associated to the "redback" PSR J2215+5135 [97].

Unlike mass, radii observations from NS thermal emission or X-ray burst is not very accurate [6, 98–103]. A clear understanding of the composition of the atmosphere, the magnetic field, the NS spinning as well as the density of the interstellar medium is however necessary to get reliable estimation of the NS radius [6]. Model dependence is also an important source of uncertainties since it was shown that it can shift radii up to 2 km [102]. Eventually, the predicted radii from the X-ray observations for $M = 1.4M_{\odot}$ is in the 9.8 – 11 km range. However, it could also be as small as $R_{1.4} = 8.9_{-1.0}^{+1.7}$ km from Ref. [102] or as large as $R_{1.4} = 12_{-0.45}^{+0.66}$ km from Refs. [98, 99]. In a recent work, a semi-agnostic meta-model identical to ours was directly injected in the analysis of the thermal emission from 7 qLMXB [103]. The constant radius approximation of Ref. [104] was also performed with the new data, providing a radius of about $R_{NS} \approx 11.06 \pm 0.4$ km. Injecting constraints from nuclear physics and neglecting possible phase transitions in dense matter, the radius of a $1.4M_{\odot}$ NS is predicted to be $R_{1.4} \approx 12.4 \pm 0.4$ km. The observation of a NS with a lower radius would clearly indicate a softening of the EoS induced by new degrees of freedom which are not contained in our nuclear physics meta-modeling.

Now let us start to give some technical details. Since NS are formed from the balance between pressure and gravity, it is fundamental to define a term named as equation of state (EoS). This "EoS" represents the dependence of the pressure P on mass density ρ (associated energy density $\varepsilon = \rho c^2$) and on temperature T of matter where ρ is source of gravity, and P is governed by the interactions between the particles. Approximately 1 minute after the explosion, NS reaches a thermal equilibrium between the core and the crust in which its core temperature is around 0.03–0.12 MeV [105]. Temperature T therefore can be neglected on EoS at first order. The model dependency comes from $P(\rho)$ and the related interactions. Therefore, theoretical predictions of models and interactions from nuclear physics cannot accurately constrain observational measurements. More precisely, the extrapolations to $\rho = 2 - 10\rho_{\text{sat}}$ is still under discussion since the behavior of EoS for $\rho \geq \rho_{\text{sat}}$ is not known from nuclear experiments. Therefore, more precise observations are needed to understand the high density behavior of the EoS.

Another debated topic about the high density behavior of NS EoS is the phase transition. For densities above $\rho \sim 3\rho_{\text{sat}}$, new degrees of freedom could appear, such as pion condensation [20–22], hyperonization [9–13, 15] or phase transition to quark matter [16–19]. In general the occurrence of new degrees of freedom tends to soften the EoS, and thus reduce the radius, except in the case of the quarkyonic model, which describe the transition to quark matter as a crossover [19]. However, the softening is smaller at second order phase transition such as hyperonization and pion condensation compared to first order one such as quark matter. In addition, it is also possible to consider pion condensation as a first order phase transition, which dramatically softens the NS EoS (see Ref. [20] for details). A recent article (Ref. [22]) discuss a possibility for pion condensation in nuclear medium by extracting Landau-Migdal parameter from Gamow-Teller transitions of ^{132}Sn to ^{132}Sb . However, the effect of pion condensation on EoS is predicted to be negligible from nuclear matter calculations due to the low energy gap of superconductivity [21]. Therefore, other kind of phase transitions are subject of debate: Hyperonization and quark matter. At large densities, a substantial population of hyperons are expected because the Fermi energy of neutrons becomes of the order of their rest mass, leading to an increase of the hyperon fraction, but it also reduces the degeneracy pressure inside the NS, leading to soft NS EoS at high densities. The softening of the EoS also reduces the maximum mass, and therefore the observation constraint for $M_{\text{max}}^{\text{obs}}$ could be crucial for the EoS selection. This is often referred as the hyperonization issue [9–11, 13]. On the other hand, phase transition to deconfined quark matter could also soften the NS EoS without breaking down the $M_{\text{max}}^{\text{obs}}$ constrain, if the speed of sound in dense quark matter is fixed to a large value (above the conformal limit of $C_{s,\infty}^2 = 1/3$ [106]) [107–109].

4.3 A New Constraint: GW170817

As reviewed in previous discussion, the precision of observational mass/radii measurements is not enough to determine NS EoS, while nuclear experiments are only constraining the densities around saturation densities ($\rho_{\text{sat}} = 2.7 \times 10^{14} \text{ g/cm}^3$) which is far from the densities of NSs ($\rho = 3 - 10 \rho_{\text{sat}}$). Therefore, the extrapolation of these models are indeed inside the observational margins but not enough for an accurate definition of interior structure. However, there is an additional constraint which is directly connected to the interior structure of NS: Tidal deformability [27–29]. During the in-spiral, two NS in a binary emit gravitational wave while dancing with each other and during that process, each of them is deformed due to the tidal gravitational field created by their companion. The emitted gravitational wave can be understood as a composition of two different parts: The orbital motion of each individual mass and the quadrupole response of the tidal gravitational field. The quadrupole part is

important since it includes geometry/distribution of mass (energy) inside the NSs. In other words it is directly connected to the NS EoS. This kind of effect is characterized by the tidal deformability ($\tilde{\Lambda}$) which will be detailed in Sec. 4.7.

In 2017, the first gravitational waves (GW) from a binary NS merger (GW170817) have been detected by the LIGO-Virgo collaboration [25, 26], providing an estimation of the NS tidal deformability $\tilde{\Lambda}$. The tidal deformability is similar to the measure of compactness of star [25], and together with a measure of the mass, it can be used to extract the NS radius [109]. The tidal deformability extracted from GW170817 is $70 < \tilde{\Lambda} < 720$ at 90% confidence level from Ref. [26], and $70 < \tilde{\Lambda} < 500$ from Ref. [30]. Moreover the $\tilde{\Lambda}$ probability distribution function (PDF) exhibit an interesting structure, doubly peaked in Ref. [26] (with a large and a small peak) and only single peaked in Ref. [30].

After the detection, the GW170817 signal has been confronted to various nuclear models of EoS, going from the most agnostic ones, such as piece-wise polytropes [110–113] and sound speed EoS [108, 109], semi-agnostic approaches where matter composition is known, Taylor-Expanded EoS [108, 109, 114–116]) or more traditional approaches based on nuclear interactions or Lagrangian, such as Skyrme-Type Functional [115, 117–120], and Relativistic Mean Field [115, 117, 121–123]. In Refs. [121, 122], based on Relativistic Mean Field modeling, the authors concluded that the nuclear empirical parameters L_{sym} is independent from the radius at $1.4M_{\odot}$ and that most of the explored EoSs are inside the tidal deformability limit ($\tilde{\Lambda} < 720$). In Refs [118] and [120], 5 and 28 Skyrme Type Functionals were analyzed predicting NS radii to be $11.8 \leq R_{1.4} \leq 12.8$ km from Ref. [118] ($R_{1.4} = 11.6 \pm 1$ km from Ref. [120]) and the tidal deformability for canonic NS mass ($1.4M_{\odot}$) $308 < \Lambda_{1.4} < 583$, respectively. Additionally, it is suggested that the nuclear isoscalar giant monopole resonance (ISGMR) could constrain the compactness of NS [120]. This result is important since we also use ISGMR to constrain NS EoS. In Refs. [111, 112], polytropes were used to calculate NS EoS leading to $12 \leq R_{1.4} \leq 13.7$ km for the canonical $1.4M_{\odot}$ NS radius. Similar results are found using both Relativistic Mean Field and Skyrme Type Functionals [114, 115, 117], as well as Taylor-Expanded EoS [114, 115]. Contrary to Ref. [121], Taylor-Expanded EoS from Ref [115] showed that the tidal deformability has a specific impact on both the slope of the incompressibility at the saturation density M_0 and L_{sym} (see Eq. 4.4). Recently, GW170817 has been reanalysed based on an agnostic approach and including a constraint on the maximal mass of NS [124]. This analysis concluded that the NS radius shall be $R \sim 11 \pm 1$ km.

In addition to the GW signal, the GW170817 binary NS merger have produced an

observed electromagnetic signal (AT2017gfo) and a gamma-ray burst (GRB170817A). These additional signals are influenced by the properties of the in-spiral NS, and could potentially also help the characterization of the tidal deformability. A recent multi-messenger Bayesian analysis have been performed based on the present knowledge and modeling of the EM and GRB signals [31]. This analysis has considerably reduced the Bayesian probability corresponding to $\tilde{\Lambda} \leq 300$, which incidentally reinforce the probability for $\tilde{\Lambda} \geq 300$ [31]. While one should expect improved modeling of the electromagnetic and gamma-ray burst emission before rising strong conclusions, this analysis illustrates how a global understanding of the transient event could shed light on the estimation of the tidal deformability.

4.4 Main Principles

Until now we provided observational and theoretical data about NSs, which also describes the starting point and framework of our work. Therefore, it is worth to mention about the principle of this work before explaining theoretical foundations. As we discussed above, the most common description of the NS is a giant nuclear system which contains neutrons and protons, as well as a gas of electrons and muons at the β equilibrium [2]. If we assume this description is true, it opens a possibility to define high density region of nuclear EoS from the NS observations. Hence, since we aim at exploring the limits of nucleonic hypothesis for the composition of the core of NS, we currently did not include phase transition at high density. All in all, the NS EoS should satisfy following conditions:

- The EoS should have compatible results at finite nuclear properties and its implications should be feasible to the nuclear physics without model dependency.
- The EoS should satisfy to the NS observations in terms of maximum observed NS mass and tidal deformability.
- The EoS should naturally satisfy causality and stability conditions at all densities [125].

4.5 A Semi-Agnostic Approach: The Meta-Model

4.5.1 Meta-Model: Introduction

We consider an semi-agnostic approach which is mainly parameterized in terms of nuclear empirical parameters (describing EoS fundamental properties such as the nuclear incompressibility) and can thus be easily related to experimental knowledge

from nuclear physics. At variance to fully agnostic approaches such as piece-wise polytropes [110–113] or sound speed model [108, 109], the present meta-model can predict proton, electron and muons ratios as function of the density. These ratios are controlled by the density dependence of the symmetry energy, and therefore the meta-model establishes correlations between particle ratios and nuclear empirical parameters. It allows to follow the β equilibrium and any path out-of-equilibrium, such as the ones encountered in supernovae core collapse [37]. Hence, it can reproduce the EoSs based on any nuclear interactions such as Skyrme-Type Functional or Relativistic Mean Field by choosing a unique empirical parameter set. Thanks to the meta-model, the problem reduces to find empirical parameters rather than working either on interaction or framework.

Lets us explain how the meta-model can potentially include all EoSs from nuclear physics. To do so, one should perform a link between experimental nuclear observables and theory, which can be done by nuclear empirical parameters. The link between NS matter and nuclear experiments can be performed through the nuclear empirical parameters, directly connected to the properties of the EoS. These parameters are defined as the Taylor coefficients of the binding energy density for symmetric matter (SM) e_{sat} and for the symmetry energy e_{sym} ,

$$e_{\text{sat}}(n_0) = E_{\text{sat}} + \frac{1}{2}K_{\text{sat}}x^2 + \frac{1}{3!}Q_{\text{sat}}x^3 + \frac{1}{4!}Z_{\text{sat}}x^4 + O(x^5), \quad (4.4)$$

$$e_{\text{sym}}(n_0) = E_{\text{sym}} + L_{\text{sym}}x + \frac{1}{2}K_{\text{sym}}x^2 + \frac{1}{3!}Q_{\text{sym}}x^3 + \frac{1}{4!}Z_{\text{sym}}x^4 + O(x^5). \quad (4.5)$$

where the Taylor expansion parameter is $x = (n_0 - n_{\text{sat}})/(3n_{\text{sat}})$ [126], n_0 being the isoscalar density for protons and neutrons, $n_0 = n_n + n_p$. In the equation above, E_{sat} is the saturation energy, K_{sat} is the incompressibility modulus, Q_{sat} is the skewness parameter, E_{sym} is the symmetry energy, L_{sym} is the slope of the symmetry energy, K_{sym} is the curvature of the symmetry energy sometimes called the symmetry incompressibility and Q_{sym} is the third derivative of the symmetry energy. Besides, the parameter Z_{sat} (Z_{sym}) is the fourth derivative of saturation (symmetry) energy which is completely unknown. Assuming that these two quantities (e_{sat} and e_{sym}) are the leading ones, the binding energy in isospin asymmetric matter (AM) can be expressed as,

$$e(n_0, n_1) = e_{\text{sat}}(n_0) + \left(\frac{n_1}{n_0}\right)^2 e_{\text{sym}}(n_0), \quad (4.6)$$

where the isovector density is defined as $n_1 = n_n - n_p$. Note that Eq. (4.6) neglects

the small contribution beyond the quadratic terms in isospin asymmetry.

4.5.2 Enhanced Fermi Gas Calculation

Let us now turn to the meta-model. As it is well known, neutrons and protons can be considered as independent particles in Fermi Gas (FG) approximation. Since the FG can be considered only kinetic contribution of particles, one could make an extension by including interactions. Let us start to explain the FG model, before meta-modeling. The kinetic energy of FG can be expressed as:

$$T_n = \frac{\langle P^2 \rangle}{2M_n} = \left(\frac{3}{5}\right) \frac{P_F^2}{2M_n}, \quad (4.7)$$

where M_n is the mass of nucleons ($n=n$ for neutrons or p for protons), P and P_F are the momentum and the Fermi momentum respectively. In the case of AM, the total kinetic energy can be written as

$$T(p, n, T_p, T_n) = pT_p + nT_n. \quad (4.8)$$

Here T_p (T_n) is the kinetic energy for proton (neutron) and p (n) is proton (neutron) number. However, one needs to relate p (n) to n_0 and n_1 . This can be done by

$$n_p = \frac{p}{\Omega}, \quad (4.9)$$

$$n_n = \frac{n}{\Omega}, \quad (4.10)$$

where Ω is the total volume. Since the volume is unchangeably large in the nuclear matter concept, we do not use energy but rather energy density as a measurable quantity. We then define the kinetic energy density instead kinetic energy:

$$t^{FG*}(n_p, n_n, T_p, T_n) \equiv \frac{T(p, n, T_p, T_n)}{\Omega} = n_p T_p + n_n T_n. \quad (4.11)$$

On the other hand, the neutron and proton densities, n_n and n_p , can be defined as

$$n_{n/p} = \frac{1}{3\pi^2} k_{F_{n/p}}^3, \quad (4.12)$$

in terms of the Fermi momentum $k_{F_{n/p}}$. Using Eq. (4.12) to Eq. (4.11) in terms of n_0 and n_1 , one can obtain to kinetic energy density for isospin asymmetry as

$$t^{FG*}(n_0, n_1) = \frac{t_{\text{sat}}^{FG}}{2} \left(\frac{n_0}{n_{\text{sat}}}\right)^{2/3} f_1(\delta) \quad (4.13)$$

where $t_{\text{sat}}^{FG} = 3\hbar^2/(10m)(3\pi^2/2)^{2/3}n_{\text{sat}}^{2/3}$ is the kinetic energy per nucleons in SM and at saturation, m is nucleonic mass taken identical for neutrons and protons ($m = (m_n + m_p)/2 = 938.919 \text{ MeV}/c^2$), giving $t_{\text{sat}}^{FG} = 22.1 \text{ MeV}$. The function $f_1(\delta)$ is defined as

$$f_1(\delta) \equiv (1 + \delta)^{5/3} + (1 - \delta)^{5/3}, \quad (4.14)$$

where, $\delta = (n_1/n_0)$ is called isospin asymmetry parameter. The two boundaries $\delta = 0$ and 1 correspond to SM and to neutron matter (NM), respectively, while any value of δ between -1 and 1 defines AM.

Let us focus on look the Landau effective mass. The effective mass is a useful concept used to characterize the propagation of quasiparticles inside a strongly interacting medium, such as nuclei or nuclear matter. It reflects the nonlocality in space and time of the quasiparticle self-energy. The nonlocality in space, also called the Landau effective mass, is related to the momentum dependence of the nuclear interaction. The Landau effective mass depends on the isoscalar and isovector densities and can be different for neutrons and protons. The Landau effective mass can also be different for SM and AM: $m_n^* = m_p^*$ for SM whereas in AM, the neutron and proton Landau effective mass can be different. Therefore the isospin splitting of the Landau effective mass can then be generally expressed as

$$\Delta m^*(n_0, n_1) = m_n^*(n_0, n_1) - m_p^*(n_0, n_1). \quad (4.15)$$

Two quantities ($m_n^*(n_0, n_1)$ and $m_p^*(n_0, n_1)$) can be written as an expansion for SM at saturation m_{sat}^* and the isospin splitting taken for $n_0 = n_1 = n_{\text{sat}}$ in NM, Δm_{sat}^* . The momentum dependence of the nuclear interaction gives rise to the concept of effective mass: An average effect of the in-medium nuclear interaction is to modify the inertial mass of the nucleons. Thereafter the Landau effective mass can be parameterized according to isospin asymmetry in the following way;

$$\frac{m}{m_{\tau}^*(n_0, n_1)} = 1 + (\kappa_{\text{sat}} + \tau_3 \kappa_{\text{sym}} \delta) \frac{n_0}{n_{\text{sat}}} \quad (4.16)$$

where $\tau_3 = 1$ for neutrons and -1 for protons (τ is proton and neutron), and the parameters κ_{sat} and κ_{sym} are a function of m_{sat}^* and Δm_{sat}^* . In the framework of the meta-model, we use m_{sat}^* and Δm_{sat}^* as inputs as well as the empirical parameters defined in Eq. (4.4) (the connection to empirical parameters will be shown explicitly in this section). In Eq. 4.16, both κ_{sat} and κ_{sym} can be rewritten from m_{sat}^* and Δm_{sat}^*

at $n = n_{\text{sat}}$:

$$\begin{aligned}\kappa_{\text{sat}} &= \frac{m}{m_{\text{sat}}^*} - 1 = \kappa_s, \text{ in SM } (\delta = 0), \\ \kappa_{\text{sym}} &= \frac{1}{2} \left[\frac{m}{m_n^*} - \frac{m}{m_p^*} \right] = \kappa_s - \kappa_v, \text{ in NM } (\delta = 1).\end{aligned}\quad (4.17)$$

Thereafter, one can write m from $m_p^*(n_0, n_1)$ and $m_n^*(n_0, n_1)$ at $n_0 = n_1 = n_{\text{sat}}$ with the help of Eq. (4.16) to the Eq. (4.13), the new expression for the kinetic energy in nuclear matter appears which includes effective mass parameters:

$$\begin{aligned}t^{FG*}(n_0, n_1) &= \frac{t_{\text{sat}}^{FG}}{2} \left(\frac{n_0}{n_{\text{sat}}} \right)^{2/3} \left[\left(1 + \kappa_{\text{sat}} \frac{n_0}{n_{\text{sat}}} \right) f_1(\delta) \right. \\ &\quad \left. + \kappa_{\text{sym}} \frac{n_0}{n_{\text{sat}}} f_2(\delta) \right],\end{aligned}\quad (4.18)$$

where the new function f_2 is defined as

$$f_2(\delta) \equiv \delta(1 + \delta)^{5/3} - \delta(1 - \delta)^{5/3}. \quad (4.19)$$

4.5.3 Meta-Eos: Linking Nuclear EoS Parameters to the Model

In this work, we consider the metamodeling ELFc introduced in Ref. [37]. In this metamodeling, the energy per particle is defined as

$$e(n_0, n_1) = t^{FG*}(n_0, n_1) + v(n_0, n_1). \quad (4.20)$$

The first term is the kinetic energy density as presented in Eq. (4.18) and the second term is the interaction potential which can be expressed as

$$v(n_0, n_1) = \sum_{a \geq 0}^N \frac{1}{a!} (c_a^{\text{sat}} + c_a^{\text{sym}} \delta^2) x^a u_a(x), \quad (4.21)$$

where $u_a(x) = 1 - (-3x)^{N+1-a} \exp(-bn_0/n_{\text{sat}})$ and b is fixed to be $b = 10 \ln 2 \approx 6.93$. The function $u_a(x)$ and the parameter b are fitted according to the high density behavior of nuclear EoSs which is generated from various models [37].

Fixing parameters κ_{sat} and κ_{sym} , the coefficients c_a^{sat} and c_a^{sym} are directly related to the empirical parameters through the following one-to-one correspondences by

comparing Eq. (4.20) and Eq. (4.4),

$$\begin{aligned}
c_{a=0}^{sat} &= E_{\text{sat}} - t_{\text{sat}}^{FG}(1 + \kappa_{\text{sat}}), \\
c_{a=1}^{sat} &= -t_{\text{sat}}^{FG}(2 + 5\kappa_{\text{sat}}), \\
c_{a=2}^{sat} &= K_{\text{sat}} - 2t_{\text{sat}}^{FG}(-1 + 5\kappa_{\text{sat}}), \\
c_{a=3}^{sat} &= Q_{\text{sat}} - 2t_{\text{sat}}^{FG}(4 - 5\kappa_{\text{sat}}), \\
c_{a=4}^{sat} &= Z_{\text{sat}} - 8t_{\text{sat}}^{FG}(-7 + 5\kappa_{\text{sat}}),
\end{aligned} \tag{4.22}$$

and

$$\begin{aligned}
c_{a=0}^{sym} &= E_{\text{sym}} - \frac{5}{9}t_{\text{sat}}^{FG}[1 + (\kappa_{\text{sat}} + 3\kappa_{\text{sym}})], \\
c_{a=1}^{sym} &= L_{\text{sym}} - \frac{5}{9}t_{\text{sat}}^{FG}[2 + 5(\kappa_{\text{sat}} + 3\kappa_{\text{sym}})], \\
c_{a=2}^{sym} &= K_{\text{sym}} - \frac{10}{9}t_{\text{sat}}^{FG}[-1 + 5(\kappa_{\text{sat}} + 3\kappa_{\text{sym}})], \\
c_{a=3}^{sym} &= Q_{\text{sym}} - \frac{10}{9}t_{\text{sat}}^{FG}[4 - 5(\kappa_{\text{sat}} + 3\kappa_{\text{sym}})], \\
c_{a=4}^{sym} &= Z_{\text{sym}} - \frac{40}{9}t_{\text{sat}}^{FG}[-7 + 5(\kappa_{\text{sat}} + 3\kappa_{\text{sym}})].
\end{aligned} \tag{4.23}$$

The one-to-one correspondence between the meta-model coefficients c_a^{sat} , c_a^{sym} and the empirical parameters directly bridges the analysis of the impact of the empirical parameters on the properties of the equation of state to the predictions of NS properties. To summarise, we got 10 empirical parameters from the Taylor expansion (E_{sat} , n_{sat} , K_{sat} , Q_{sat} , Z_{sat} , E_{sym} , L_{sym} , K_{sym} , Q_{sym} , and Z_{sym}) and 2 parameters from Landau effective mass (m_{sat}^* and Δm_{sat}^*) which are inputs of the metamodel.

4.5.4 The Condition of β Equilibrium

Let us now discuss about the β equilibrium. Until here, we discussed how to calculate EoS if isovector (n_0) and isoscalar (n_1) densities are known. In case of NSs, these densities are determined from the β equilibrium condition: As discussed in Sec. 4.1, dying star core becomes neutron rich nuclear matter with electron capture processes. However, 1 minute after the supernovae explosion, NSs begin to reach thermal equilibrium where electron capture process and β decay occur at a same rate. At this point particle fraction will no longer change and the EoS will become stabilized [105]. Therefore, NS EoS can be investigated as β equilibrium nuclear matter. Given a proton fraction $x_p = n_p/n_0$, charge neutrality condition immediately imposes $x_p = x_e$ where x_e is the electron fraction and we also have $p_{F_e} = p_{F_p}$ for Fermi momenta. The impact of the β equilibrium condition can easily be calculated by minimizing the total

energy of the system:

$$\varepsilon(n_p, n_n) = e(n_p, n_n) + E_{\text{elec}} + \frac{n_p}{n_0} m_p c^2 + \frac{n_n}{n_0} m_n c^2, \quad (4.24)$$

where m_p (m_n) are the rest mass of proton (neutron), $e(n_p, n_n)$ the energy density from the meta-model and E_{elec} the electron energy density which can be written from the ultra-relativistic limit:

$$E_{\text{elec}} = c(p_{F_e}^2 + m_e^2 c^2)^{1/2}, \quad (4.25)$$

where p_{F_e} is the Fermi momentum, c speed of light and m_e the rest mass of electron. One then can calculate each particle fraction by minimizing the total energy (see Ref. [127] for details):

$$\frac{d\varepsilon(n_p, n_n)}{dx_p} = 0. \quad (4.26)$$

The advantage of the meta-model is that it is analytical, fast computed, very flexible and can reproduce most of existing nucleonic EoS. Moreover, it keeps information concerning matter composition, such as the neutron/proton ratio, the fraction of electrons and muons. It is therefore optimal for extensive statistical analyses which require the set-up of a large number of EoS samples. In the next sections, we briefly detail how the NS properties such as masses, radii and tidal deformabilities can be related to the nuclear equation of state assuming general relativity (TOV and pulsation equation) [3, 27–29, 38, 39].

4.6 Building a Neutron Star: Tollman-Oppenheimer-Volkoff Equations

According to Birkhoff's theorem the Schwarzschild solution is the most general description outside a nonrotating, spherically symmetric star [8]. Although, the Schwarzschild solution works well outside spherical objects, one need to define a more general metric to describe energy and pressure profiles inside the NS. Starting time-independent form of the Schwarzschild solution, $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -e^{2\Phi(r)} c^2 dt^2 + e^{2\lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$, where the potential $\Phi(r)$ and the function $\lambda(r)$ only depend on r (the function λ being fixed to $e^{-2\lambda} = 1 - 2Gm/(c^2 r)$), the Einstein equation produce the necessary relations to describe the hydrostatic equilibrium in the NS. Let us first write the Einstein equation as:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \quad (4.27)$$

where G is the gravitational constant and c the speed of light in the vacuum. The general approach for solving Einstein equation is equalling the left hand side (Einstein Tensor) and the right hand side (energy-momentum tensor). The energy-momentum tensor can be written as

$$T^{\mu\nu} = (\rho + P)u^\mu u^\nu + P g^{\mu\nu}, \quad (4.28)$$

where P the pressure and ρ the mass-energy density containing contributions from the nucleon rest mass (m_N) and from the total energy per particles (e): $\rho c^2 = (m_N c^2 + e)n_0$. Besides one can obtain diagonal components of 4-velocity by using $-c^2 = g_{\mu\nu}u^\mu u^\nu$. Equalling the left hand side and the right hand side the $G_{tt} = \frac{8\pi G}{c^4}T_{tt}$ component of Eq. (4.27):

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r), \quad (4.29)$$

where $m(r)$ the enclosed mass at radius r .

From the $G_{rr} = \frac{8\pi G}{c^4}T_{rr}$ component, one gets

$$\frac{d\Phi(r)}{dr} = \frac{Gm}{c^2 r^2} \left(1 + \frac{4\pi P r^3}{m c^2} \right) \left(1 - \frac{2Gm}{r c^2} \right)^{-1}. \quad (4.30)$$

However, a differential equation for the pressure is also required. This is done by using conservation of energy, implying that the divergence of the stress-energy tensor vanishes. Since the system is spherical symmetric, this can be done by using radial component ($\mu = r$):

$$0 = \nabla_\nu T^{r\nu} = \frac{\partial T^{r\nu}}{\partial x^\nu} + T^{\sigma\nu} \Gamma_{\sigma\nu}^r + T^{r\sigma} \Gamma_{\sigma\nu}^\nu, \quad (4.31)$$

eventually leading to

$$\frac{dP(r)}{dr} = -\rho c^2 \left(1 + \frac{P}{\rho c^2} \right) \frac{d\Phi(r)}{dr}. \quad (4.32)$$

Eqs.(4.29, 4.30 and 4.32) are named as the Tolman-Oppenheimer-Volkoff (TOV) equations [8, 38, 39]. For convenience we show these equations all together:

$$\begin{aligned} \frac{dm(r)}{dr} &= 4\pi r^2 \rho(r), \\ \frac{dP(r)}{dr} &= -\rho c^2 \left(1 + \frac{P}{\rho c^2} \right) \frac{d\Phi(r)}{dr}, \\ \frac{d\Phi(r)}{dr} &= \frac{Gm}{c^2 r^2} \left(1 + \frac{4\pi P r^3}{m c^2} \right) \left(1 - \frac{2Gm}{r c^2} \right)^{-1}. \end{aligned} \quad (4.33)$$

Eqs. (4.33) are integrated in coordinate space starting from 0 to the radius R , fixing the boundary conditions $m(0) = 0$ and $P(0) = P_c$ where $P_c(\rho = \rho_c)$ is arbitrarily varied. The pressure P decreases from the center to the surface and the NS radius is defined as the coordinate for which the condition $P(r = R) = 0$ is reached. The family of solutions with unique mass $m(R) = M$ and radii R are generated by varying the central density ρ_c .

Since there are three equations for four variables (m , P , ρc^2 and Φ) in Eqs. (4.33), one need another equation to close the system. This additional equation is provided by the equation of state of dense matter, $P(\rho c^2)$, which is evaluated at β -equilibrium for the NS conditions. NSs are formed by a crust and a core whereas in its present form the meta-model only applies to uniform matter inside the core. Since the crust includes multiple phases, we did not make an analysis for a crust EoS as well as transition density n_{tr} . Besides, we expect that the impact of the connection between the crust and the core is small for our analysis (for more details see Ref. [128]). The core EoS is matched to the crust EoS with a cubic spline starting from an arbitrary transition density $n_{tr} = 0.1n_{sat}$ to n_{sat} . Below n_{tr} , we set crust EoS to be SLY for all core EoSs. SLY is based on the Skyrme nuclear interaction SLy4 [129], which has been applied for the crust EOS considering a compressible liquid-drop model [130]. In next section we will discuss how we to GW to our advantage to describe NS EoS.

4.7 Pulsation Equations and Gravitational Wave

4.7.1 Tidal Love Number

As discussed in Sec. 4.1, the observational constraints on the internal structure of NSs are weak and there is no direct method to measure radii of NS. X-ray observations requires to have a model for the emission itself, which can be thermal (improved black-body) or a burst, or generated by a hot spot (or several) located at the surface of the neutron star. In all these cases, in addition to general relativity, a model for the emission process is required. The art of the observation is to isolate some neutron stars for which there is almost no ambiguity in the interpretation of the data, for instance see Ref. [103] for details. In the latter case, one could still argue that there is a 1–1.5 km uncertainty coming from systematical uncertainties (model dependence). While it is not entirely satisfactory, it is comparable with the uncertainty from binary neutron star mergers. Thus, binary neutron star mergers are presently not much better in the constrains it provides, but it is a promising new observable which could bring more accuracy by accumulating more events, and it could be contrasted with other methods to provide a consistent picture. However, measuring GW from NS in-spiral can provide additional constraint on the NS EoS. Coalescing binary NSs are one of the

most important sources for ground-based gravitational wave detectors [131].

One way to utilise GW to describe EoS is to make hydrodynamics simulation of NS-NS mergers by using several last orbits before the merging [132]. However, trying to extract EoS information in this way rises several difficulties which complex behavior requires solving the nonlinear equations of general relativity together with relativistic hydrodynamics. Moreover, the signal includes unknown quantities such as spins and angular momentum distribution inside the NS (see Ref. [27] for details). There is however an easier method to use GW in NS EoS favour. During the early time of in-spiral, the GW signal is very clean and the influence of tidal effects correspond the only small correction to waveform's phase. However, at the late times of merger, it could corrupt GW signal, or alternatively could give information about the NS structure. This has been studied by several authors (see Ref. [133] and therein). The influence of the internal structure of the NS on the gravitational wave phase in this early regime of the in-spiral is characterized by a single parameter, namely, the ratio λ of the induced quadrupole to the perturbing tidal field. This ratio λ (or Λ see Eq. (4.49)) also called tidal deformability, is related to the star's tidal Love number k_2 by

$$\lambda = \frac{2k_2}{3R^5} \quad (4.34)$$

where R the radius of NS [27, 28].

4.7.2 Pulsation Equations

The tidal Love number k_2 is an outside solution of linearized perturbation equations due to an external quadrupolar tidal field. To derive the expression of k_2 , we first express the Einstein equation in the effect of a quadrupolar tidal field. We use derivation from Ref. [134] which uses spherical harmonics in Regge-Wheeler gauge for the electrical part of the even-parity static pulsation. In the presence of a quadrupole field, the metric can be described by making first order perturbation to the Schwarzschild metric:

$$g_{\mu\nu}^{(ac)} = g_{\mu\nu} + h_{\mu\nu}, \quad (4.35)$$

where $h_{\mu\nu}$ is a linearized metric perturbation and $g_{\mu\nu}$ is the Schwarzschild metric, which is $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -e^{2\Phi(r)}c^2dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$. Using the Regge-Wheeler gauge condition, $h_{\mu\nu}$ becomes

$$h_{\mu\nu} = [-e^{2\Phi(r)}H_0(r)c^2dt^2 + e^{2\lambda(r)}H_2(r)dr^2 + r^2K(r)(d\theta^2 + \sin^2\theta d\phi^2)]Y_{20}, \quad (4.36)$$

where Y_{20} is the spherical harmonic. It should be noted that non-diagonal elements of both $g_{\mu\nu}$ and $h_{\mu\nu}$ are zero. In order to derive $H_0(r)$, $H_2(r)$ and $K(r)$ functions, one

needs to use the linearized version of Einstein equation. Therefore, before finding metric elements, we first discuss how to derive linearized Einstein equation. Thus is obtained by taking variations of both side of Einstein equation:

$$\delta G_\nu^\mu = \frac{8\pi G}{c^4} \delta T_\nu^\mu. \quad (4.37)$$

The left hand side is:

$$\delta G_{\mu\nu} = \delta[R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}] = \delta R_{\mu\nu} - \frac{1}{2}[h_{\mu\nu}R + g_{\mu\nu}\delta R], \quad (4.38)$$

where $\delta R = g^{\mu\nu}\delta R_{\mu\nu} + h^{\mu\nu}R_{\mu\nu}$. It is convenient to use δG_ν^μ instead of $\delta G_{\mu\nu}$ since the equations are much simpler in this way. The corresponding form of Einstein tensor is

$$\delta G_\nu^\mu = \delta(g^{\mu\beta}G_{\beta\nu}) = h^{\mu\beta}G_{\beta\nu} + g^{\mu\beta}\delta G_{\beta\nu}, \quad (4.39)$$

where $G_{\beta\nu} = R_{\beta\nu} - \frac{1}{2}Rg_{\beta\nu}$ is the unperturbed Einstein tensor based on the Schwarzschild metric. The right hand side of linearized Einstein equation is based on the perturbed energy momentum tensor which can be written as [134],

$$\delta T_\nu^\mu = \begin{cases} \delta T_\nu^\mu = \delta\rho, & \mu = \nu = t; \\ \delta T_\nu^\mu = -\delta P, & \mu = \nu \text{ and } \mu = r, \theta, \phi; \\ \delta T_\nu^\mu = 0, & \mu \neq \nu. \end{cases} \quad (4.40)$$

Equalling left hand side and right hand side of Eq. (4.37), one can express the variables $H_0(r)$, $H_2(r)$ and $K(r)$ in terms of $H_0(r) = H(r)$ as

$$\delta G_\theta^\theta - \delta G_\phi^\phi = -\delta P + \delta P = 0 \implies -H_2(r) = H_0(r) = H(r), \quad (4.41)$$

$$\delta G_\theta^r = 0 \implies \frac{dK(r)}{dr} = 2H(r)\frac{d\Phi(r)}{dr} + \frac{dH(r)}{dr}, \quad (4.42)$$

where $d\Phi(r)/dr$ is already defined in Eq. (4.33). Besides, there are still variables coming from the perturbed energy momentum tensor (δP and $\delta\rho$). However, it can be elegantly process by using $\delta\rho = c_s\delta P$ where $c_s \equiv (dP/d\rho)^{-1}$ is the speed of sound. Keeping this mind, one can replace δP as

$$\delta P = -c^4 \frac{\delta G_\theta^\theta + \delta G_\phi^\phi}{16\pi G}, \quad (4.43)$$

using in to

$$\delta G_t^t - \delta G_r^r = \frac{8\pi G}{c^4} \delta P (c_s + 1), \quad (4.44)$$

leads to the pulsation equation for NS interior;

$$r \frac{dy(r)}{dr} + y(r)^2 + y(r)F(r) + Q(r) = 0, \quad (4.45)$$

with

$$\begin{aligned} F(r) &= \frac{1}{r - 2Gm/c^2} \left(\frac{r + 4\pi Gr^3}{P - \rho c^2} \right), \\ Q(r) &= \frac{4\pi Gr^3/c^2}{r - 2Gm/c^2} \left(5\rho + \frac{9P}{c^2} + \frac{P + \rho c^2}{\rho c_s} \right) \\ &\quad - \frac{4\pi Gr^3/c^2}{r - 2Gm/c^2} \left(\frac{6}{4\pi Gr^2/c^2} \right) - \left(\frac{2G^2 r}{c^4} \right) \\ &\quad \times \left(\frac{m + 4\pi r^3 P/c^2}{r - 2Gm/c^2} \right)^2, \end{aligned} \quad (4.46)$$

where $y(r) \equiv RH'(r)/H(r)$ and $H' = dH/dr$. However, Eq. (4.45) is only valid inside of the NS. Outside of the NS, Eq. (4.45) becomes associated Legendre equation with $l = m = 2$ for which $H(r)$ is analytically solvable.

4.7.3 An Observable: Tidal Deformability

In order to build a bridge between the pulsation equations and tidal Love number k_2 , one can make a multipolar expansion (in the framework of general relativity) for mass, by only taking leading terms [135]:

$$-\frac{g_{tt} + 1}{2} = -\frac{m}{r} - \frac{3Q_{ij}}{2r^3} n^i n^j + \dots + \frac{R_{0i0j}}{2} r^2 n^i n^j + \dots, \quad (4.47)$$

where $Q_{ij} = -\lambda R_{0i0j}$ and $\vec{n} = (\sin(\theta)\cos(\phi), \sin(\theta)\sin(\phi), \cos(\theta))$. Using the outside solution of $H(r)$ (outside version of Eq. (4.45)) and Eq. (4.34), tidal Love number (k_2) can be extracted as:

$$\begin{aligned} k_2 &= \frac{8C^5}{5} (1 - 2C)^2 [2 + 2C(Y - 1) - Y] \\ &\quad \times \{ 2C[6 - 3Y + 3C(5Y - 8)] \\ &\quad + 4C^3[13 - 11Y + C(3Y - 2) + 2C^2(1 + Y)] \\ &\quad + 3(1 - 2C)^2 [2 - Y + 2C(Y - 1)] \\ &\quad \times \ln(1 - 2C) \}^{-1}, \end{aligned} \quad (4.48)$$

where $Y = y(R)$ is the solution of the pulsation equation at the surface of the NS. Note that $Y = y(R)$ is a continuous quantity which is valid for both inside and outside of the NS. The pulsation equation is solved once the density and pressure radial profiles are

defined from the solution of the TOV equations. Despite that the tidal Love number is proposed as a measurable quantity from GW (see Refs. [27, 28]), nowadays the mostly used related quantity is dimensionless tidal deformability (which it is still called tidal deformability) Λ [25, 30, 31]. Similarly to λ , it is defined from the tidal Love number as

$$\Lambda = \frac{2k_2}{3C^5}, \quad (4.49)$$

where $C = (GM)/(c^2R)$ is compactness for the NS of mass M and radius R .

The wave-form extracted from the LIGO-Virgo GW interferometers is in fact impacted at the fifth-order by the two-NS combined tidal deformability $\tilde{\Lambda}$, defined from each individual deformabilities of the NS, Λ_1 and Λ_2 , as

$$\tilde{\Lambda} = \frac{16}{13} \frac{(M_1 + 12M_2)M_1^4\Lambda_1 + (M_2 + 12M_1)M_2^4\Lambda_2}{(M_1 + M_2)^5}, \quad (4.50)$$

where (M_1, Λ_1) and (M_2, Λ_2) are the masses and tidal deformabilities of the individual NSs (by convention $M_1 \geq M_2$) [25]. If $M_1 = M_2$, this expression becomes $\tilde{\Lambda} = \Lambda_1 = \Lambda_2$. However, as discussed below, we shall explore the general case of asymmetric masses in our study.

4.8 Putting All Together: Bayesian Statistics

The relation between nuclear EoS empirical parameters and the NS properties is performed within the Bayesian statistical analysis. The core of the Bayesian analysis lies on Bayes theorem expressing the probability associated to a given model, represented here by its parameters $\{a_i\}$, to reproduce a set of data, $P(\{a_i\} | \text{data})$ also called the posterior PDF [40]:

$$P(\{a_i\} | \text{data}) \simeq P(\text{data} | \{a_i\}) \times P(\{a_i\}), \quad (4.51)$$

where $P(\text{data} | \{a_i\})$ is the likelihood function determined from the data comparison between the model and the measurement, and $P(\{a_i\})$ is the prior PDF which represents our knowledge or bias on the model parameters. Detailed discussions for the prior $P(\{a_i\})$ and for the likelihood probability $P(\text{data} | \{a_i\})$ are given in Sec. 4.9.1.

Table 4.1 The prior parameters (The fixed empirical parameters from group P1 and P3)

E_{sat} (MeV)	E_{sym} (MeV)	n_{sat} (fm ⁻³)	m_{sat}^*/m	$\Delta m_{\text{sat}}^*/m$	Z_{sat} (MeV)	Z_{sym} (MeV)
-15.8	32.0	0.155	0.75	0.1	0	0

The marginal one- and two-parameter probabilities are defined as [40]

$$P(a_j | \text{data}) = \left\{ \prod_{\substack{i=1 \\ i \neq j}}^5 \int da_i \right\} P(\{a_i\} | \text{data}), \quad (4.52)$$

$$P(a_j, a_k | \text{data}) = \left\{ \prod_{\substack{i=1 \\ i \neq j, k}}^5 \int da_i \right\} P(\{a_i\} | \text{data}). \quad (4.53)$$

These marginal probabilities represent the one parameter PDF and the two-parameter correlation matrix, respectively.

4.9 Constraining a Neutron Star

4.9.1 General Framework

In our analysis, we evaluate the NS EOSs for each set of empirical parameters, which are 12 free parameters in total (10 empirical parameters and two parameters associated to the Landau effective mass, see Sec.4.5.3 for details). Some of these parameters are however already well-known from the nuclear physics experiments and their small uncertainties do not impact the dense matter EoS to a large extend (see Ref. [128] for details). The 12 free parameters are therefore separated into three different groups:

- (P1) The parameters which are not varied: E_{sat} , E_{sym} , n_{sat} , m_{sat}^*/m and $\Delta m_{\text{sat}}^*/m$.
- (P2) The less-known parameters, which are varied on a uniform grid: K_{sat} , L_{sym} , K_{sym} , Q_{sat} and Q_{sym} .
- (P3) The totally unknown parameters, which however do not impact our analysis enough to be explored: Z_{sat} and Z_{sym} , since they do not play a major role for the dense matter equation of state associated to NS in the mass range between $1M_{\odot}$ and $2M_{\odot}$ which corresponds to possible masses of the binary NSs from GW170817 (see Ref. [128] for more details).

Table 4.2 The prior parameters (The empirical parameters from group (P2), which are varied on a uniform grid for two different scenarios. Changes between the two sets are indicated in bold characters. Here Min, Max are first and last values of the each parameter, Step is an increment for each iteration and N is the number of total fragment. For prior set #1 and #2, please see the text for details)

Empirical Parameters	L_{sym} (MeV)	K_{sat} (MeV)	K_{sym} (MeV)	Q_{sat} (MeV)	Q_{sym} (MeV)
Prior set #1					
Min	-10	150	-500	-1000	-2000
Max	70	280	1500	3000	2000
Step	5	10	200	400	400
N	17	14	11	11	11
Prior set #2					
Min	-10	180	-500	-1000	-2000
Max	70	280	300	3000	2000
Step	5	10	100	400	400
N	17	11	9	11	11

In Table. 4.1, we show the parameters which are not varied (from group P1), see Ref. [128] and references therein. The parameters like E_{sat} , E_{sym} and n_{sat} are well-known from finite-nuclei experiments and their uncertainty does not impact our analysis. The other parameters such as m_{sat}^*/m and $\Delta m_{\text{sat}}^*/m$ are also constrained from nuclear physics experiments, to a lower extend, but their uncertainties only weakly impact dense matter EoS [37].

In the present analysis the model parameters $\{a_i\}$ which are varied (group P2) are: L_{sym} , K_{sat} , K_{sym} , Q_{sat} and Q_{sym} . These empirical parameters are sampled on a uniform grid defined in Table 4.2. These parameters are varied between a lower (Min) and an upper (Max) value, with N steps defining a constant step. We have considered two different choices for the prior. In the prior set #1, the boundaries of the parameters are determined such that the likelihood probability reaches zero, or a very small value compared to the one inside the range. In the prior set #2, we fix the boundaries to be the ones determined from nuclear physics experiments and reported in Ref. [37], except for L_{sym} for which we allow the exploration of small values. Anticipating our results, we will show that low values for L_{sym} are favoured by the tidal deformability from GW170817. A detailed discussion about L_{sym} is made in next chapter.

The likelihood probability defines the ability of the model to reproduce the data. In the present analysis, it is defined as [40],

$$P(\text{data} | \{a_i\}) = w_{\text{filter}} \times p_{\tilde{\Lambda}} \times p_{\chi_{\text{EFT}}} \times p_{\text{ISGMR}}, \quad (4.54)$$

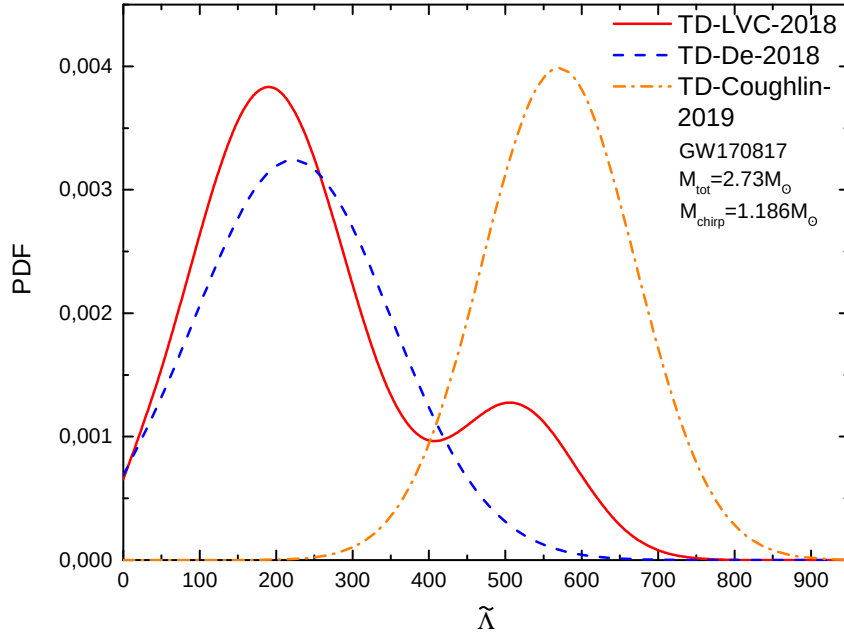


Figure 4.2 The generated likelihood functions for tidal deformability from Refs. [26, 30]

where $w_{\text{filter}}(\{a_i\})$ is a pass-band type filter which select only the models satisfying the necessary condition (C1) expressed hereafter, and the probabilities $p_{\tilde{\lambda}}$, $p_{\chi\text{EFT}}$ and p_{ISGMR} are associated to constraints (C2)-(C4) expressed hereafter. The constraints entering into the Bayesian probability (Eq. (4.54)) are:

- (C1) The necessary conditions that each viable EoS shall satisfy: causality, stability, positiveness of the symmetry energy and maximum observed mass $M_{\text{max}}^{\text{obs}}$.
- (C2) $p_{\tilde{\lambda}}$: the probability associated to the ability of the EoS to reproduce the tidal deformability extracted from the GW170817 event [26, 30].
- (C3) $p_{\chi\text{EFT}}$: the probability measuring the compatibility between the meta-model and the energy and pressure bands function of the density predicted from χ -EFT approach below saturation density [32].
- (C4) p_{ISGMR} : the probability of a given meta-model to be compatible with recent analysis of the ISGMR collective mode [33, 34].

The constraints (C1) are necessary constraints for all EoS, (C2) are constraints from astrophysics impacting high densities, while (C3) and (C4) are constraints from low-density nuclear physics. In the following, we detail how the probabilities associated to these constraints are estimated in practice.

Let us detail the constraints from the group (C1). Causality, stability and positiveness of the symmetry energy are imposed as in Ref. [128]. The constraints are imposed up to the density corresponding to the maximum density of the stable branch. We also impose that all viable EoS shall have a maximum mass $M_{\max} \geq M_{\max}^{\text{obs}} = 2M_{\odot}$ [23].

4.9.2 Constraints of the Gravitational Wave

We now come to the constraint (C2) associated to the tidal deformability from GW170817. We consider two independent GW analyses which provide two slightly different $\tilde{\Lambda}$ PDF. These PDFs are displayed in Fig. 4.2 under the legend "TD-LVC-2018" and "TD-De-2018". TD-LVC-2018 is the result of the latest analysis from the LIGO-Virgo collaboration [26] while TD-De-2018 is an independent analysis proposed in Ref. [30]. Contrary to TD-De-2018, TD-LVC-2018 has a double peak; the highest one is peaked around $\tilde{\Lambda}_{\max}^1 \approx 180$ and the smaller one is around $\tilde{\Lambda}_{\max}^2 \approx 550$. However, in TD-De-2018, the only peak is $\tilde{\Lambda}_{\max} \approx 200$. The presence of a double peak has an impact on the higher value for the 90% confidence-level: It is 720 in the case of TD-LVC-2018 while it is about 500 for TD-De-2018. Anticipating our results, the PDF from TD-De-2018 shall select more compact objects than the one assuming TD-LVC-2018.

Note that recently, a combined analysis including the electro-magnetic and GRB counterpart observations has shifted up the lower limit for $\tilde{\Lambda}$ and the centroid: $300 < \tilde{\Lambda} < 800$ also displayed on Fig. 4.2 [31].

The probability $p_{\tilde{\Lambda}}$ is calculated in the following way. For a given parameter set $\{a_i\}$, the TOV and the pulsation equations are first solved, which provides a family $\{M_i, \Lambda_i\}$, where i is an index running over the central density. We then sample the mass distribution for the two NS (M_1, M_2) by taking a set of six masses, where M_2 is distributed from $1.1M_{\odot}$ to $1.35M_{\odot}$, and M_1 is calculated such that $M_1 + M_2 = 2.73M_{\odot}$, accurately determined from GW170817. Note that eventually there are less masses in the sample if M_1 exceeds the value $M_{\max}$ for the EoS. For each sample elements the combined tidal deformability $\tilde{\Lambda}$ is calculated from Eq. (4.50) and a probability, $p_{\tilde{\Lambda}}^k$, is assigned from the PDF shown in Fig. 4.2 for the two scenarios. The final probability $p_{\tilde{\Lambda}}$ is then obtained from the averaging over sample elements,

$$p_{\tilde{\Lambda}} = \frac{1}{N} \sum_{i=k}^N p_{\tilde{\Lambda}}^k. \quad (4.55)$$

Note that there are several ways to calculate $p_{\tilde{\Lambda}}$. Another choice could have been, for instance, to assign to the parameter set the maximum probability obtained for $\tilde{\Lambda}$, $p_{\tilde{\Lambda}} =$

$\max_k p_{\tilde{\Lambda}}^k$. However, since the $\tilde{\Lambda}$ PDF only weakly depends on the mass asymmetry [30], we do not expect a large effect between these two possible prescriptions. It should also be noted that this is true since we do not consider first-order phase transitions: the mass asymmetry between the two NSs could have a strong impact on $\tilde{\Lambda}$ if the phase transition occurs at a mass in-between the ones of the two NS [109, 136].

In the present analysis, we assume that each neutron star of the binary system has the same EoS, the same particle composition and that their particle fractions is derived from the β equilibrium condition. Other exotic compositions such as Delta resonances, hyperons giving rise to Hybrid-Star/NS binaries could be considered, but are less likely.

4.9.3 Constraints of Nuclear Physics Observables

The constraint (C3) is a nuclear physics constraint which measures the proximity of the meta-model to the prediction bands for the energy per particle and the pressure in SM and NM obtained by many-body perturbation theory based on χ EFT nuclear two and three-body interactions [32], see Fig. 4.3 for illustration. At low densities, the many-body perturbation theory based on χ EFT nuclear two and three-body interactions has predicted bands based on 7 Hamiltonians which could equally well reproduce NN phase shifts and the binding energy of the deuteron [32]. These bands are represented in Fig. 4.3 together with a set of models. We compare these bands with three different models which are SLy [129], ArgonneV18 [137] and FSUGold [126]. The binding energies of these models are in good agreement with the χ EFT bands in both symmetric matter (SM) and neutron matter (NM). This is also true for the pressure in SM, but there are deviations in NM for FSUGold and SLy models, which predict the pressure above the bands for the high density region. The origin of these deviations lies in the way the χ EFT bands for the pressure is defined: It is the boundary calculated from the derivative of the binding energy predicted from the 7 Hamiltonians only. The pressure band does not exhaust all possible density dependence for the binding energy. It is therefore possible for models, such as FSUGold and SLy, to be inside the energy band and outside the pressure band. The pressure band from the χ EFT estimation provides a smaller band width than the one which would be based on all the models compatible with the energy band. It is however the width compatible with the 7 Hamiltonians that we will consider in the following. All in all, we can interpret (C3) as an common expectation of the nuclear physics, since it is calculated by using few-body observables at nucleonic scale with their theoretical uncertainties.

In practice, we estimate the following error function $\chi_{2,\chi EFT}$ for each set of

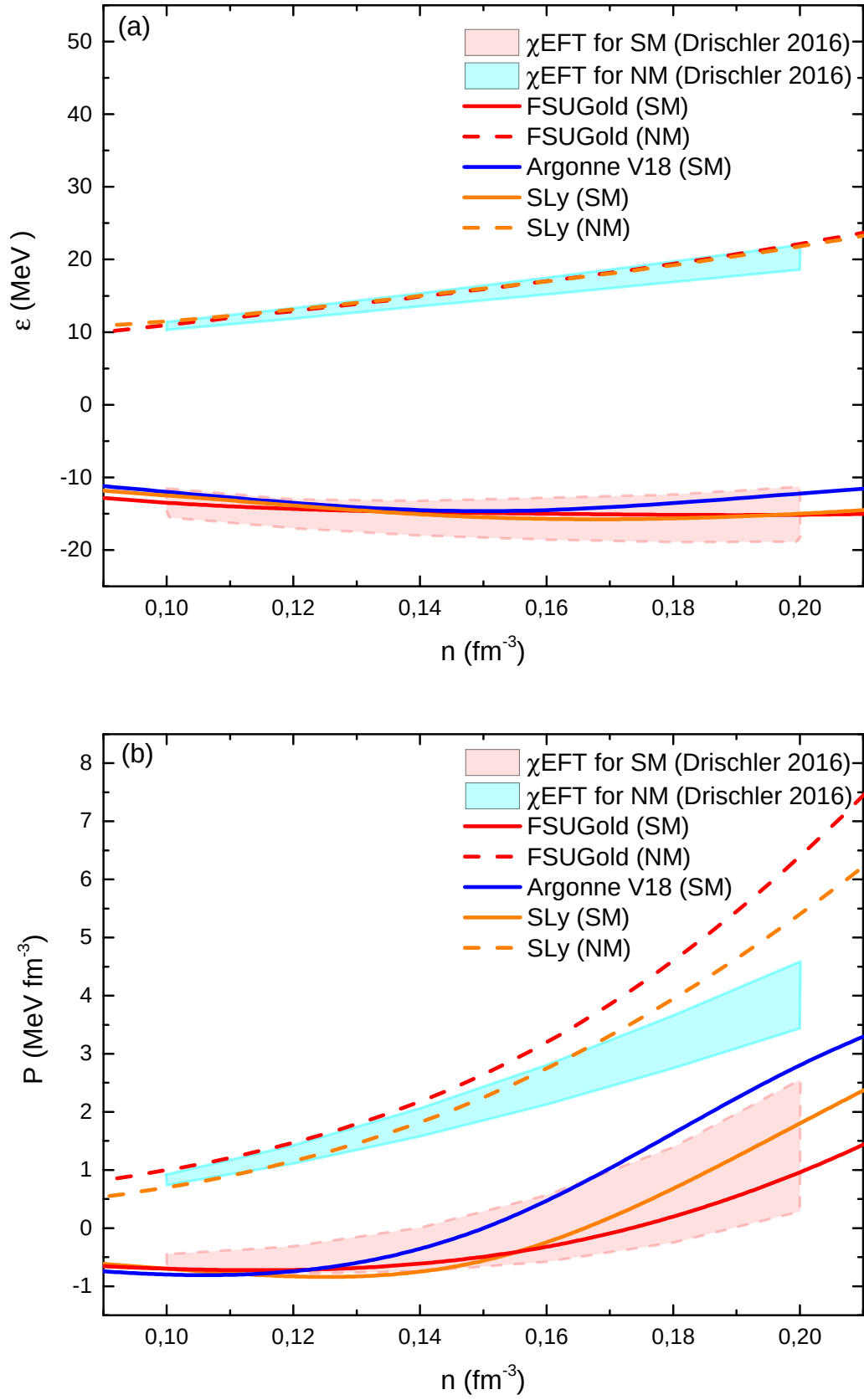


Figure 4.3 Energy (a) and Pressure (b) distributions calculated by using χEFT from the Ref. [32] for both symmetric matter (SM) and neutron matter (NM)

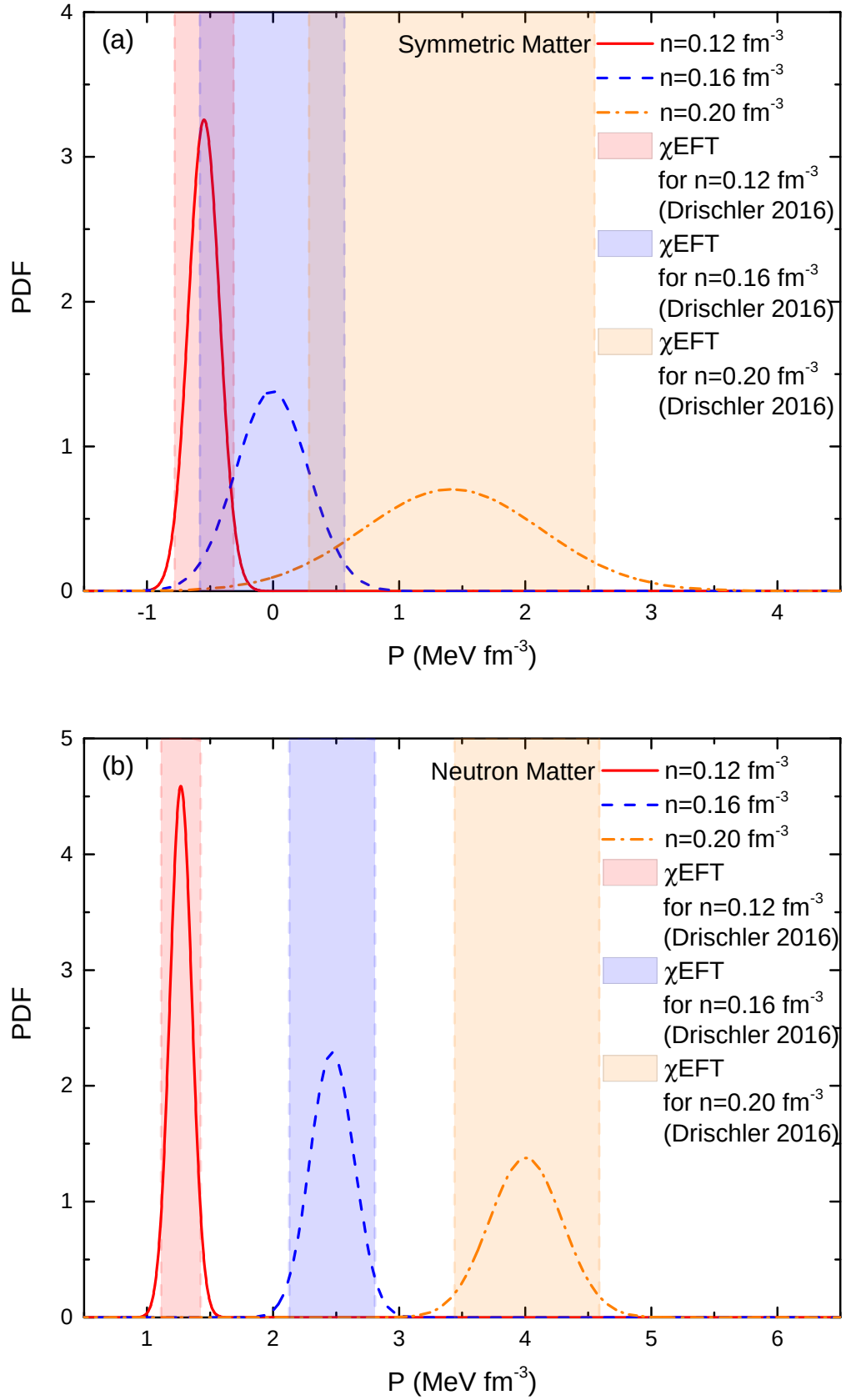


Figure 4.4 Pressure posterior functions in neutron matter (NM) (a) and symmetric matter (SM) (b) obtained from the constraint C3 associated to the χEFT bands calculated in Ref. [32]

meta-models,

$$\chi_{2,\chi EFT}^2 = \frac{1}{N_{data}} \sum_{i=1}^{N_{data}} \left(\frac{o_i^{data} - o_i(\{a_i\})}{\sigma_i} \right)^2. \quad (4.56)$$

where $N_{data} = 20$ is the number of data o_i^{data} considered here, $o_i(\{a_i\})$ is the prediction of the model and σ_i is associated to the uncertainties in the data and the accepted model dispersion. We consider 5 density points uniformly distributed between 0.12 fm^{-3} and 0.20 fm^{-3} . If Δ_i is the width of the band at each density point, we fix $\sigma_i = \Delta_i/2$ to ensure that 95% of the models lie inside the band. The small tolerance of 5% of the models outside the band is there to smoothly reduce the probability of marginal meta-models. The associated probability is thus deduced from the usual Gaussian expression,

$$p_{\chi EFT} = \exp\left(-\frac{1}{2}\chi_{2,\chi EFT}^2\right). \quad (4.57)$$

An example of likelihood function associated to the pressure for a few densities (0.12 , 0.16 and 0.20 fm^{-3}) is shown in Fig. 4.4 for SM (a) and NM (b), where only the constraint C3 is imposed. There is a nice overlap with all models inside χEFT bands with 95% confidence level (shaded regions of Fig. 4.4).

The last constrain (C4) is obtained from a recent analysis of the ISGMR in finite nuclei [33, 34]. Theoretical models designed to describe finite nuclei and applied to the calculation of the ISGMR centroid energy in ^{120}Sn and ^{208}Pb suggest that the slope of the incompressibility M_c at $n_c = 0.11 \text{ fm}^{-3}$ is well correlated to the experimental data. M_c is defined as

$$M_c = 3n_c \left. \frac{dK(n_0)}{dn_0} \right|_{n_0=n_c}, \quad (4.58)$$

where the incompressibility $K(n_0)$ in SM ($\delta = 0$) is, χ being the compressibility,

$$K(n_0) = \frac{9n_0}{\chi(n_0)} = 9n_0^2 \frac{d^2e(n_0)}{dn_0^2} + \frac{18}{n_0}P(n_0), \quad (4.59)$$

and the pressure is

$$P(n_0) = n_0^2 \frac{de(n_0)}{dn_0}, \quad (4.60)$$

It is found that $M_c = 1050 \text{ MeV} \pm 50 \text{ MeV}$ [33, 34]. The interesting feature of this parameter is that it is much less model dependent than the more frequently considered incompressibility modulus $K_{sat} = K(n_{sat})$.

In practice, we calculate the value of M_c for each of our meta-models by assigning the

following probability,

$$p_{ISGMR} = \exp \left\{ -\frac{1}{2} \left(\frac{M_c(\{a_i\}) - 1050}{25} \right)^2 \right\}, \quad (4.61)$$

where we associate the dispersion ± 50 MeV estimated in Refs. [33, 34] to the distribution of 95% of the meta-models.

4.10 Concluding Remarks

Since the model dependency is a major problem to understand universal specifications of the nuclear interactions, we used the metamodel which can reproduce all models of the nuclear physics with the help of the unique set of nuclear EoS parameters. Dealing with nuclear EoS parameters, we got 10 empirical parameters from the Taylor expansion (E_{sat} , n_{sat} , K_{sat} , Q_{sat} , Z_{sat} , E_{sym} , L_{sym} , K_{sym} , Q_{sym} , and Z_{sym}) and 2 parameters from Landau effective mass (m_{sat}^* and Δm_{sat}^*). In order to calculate NS properties and their related probabilities, we defined nuclear EoS parameters in an uniform grid by grouping them as a priori to our calculation. The first one is prior set #1 in which the boundaries of the empirical parameters are unconstrained by any background information or bias. The second one is prior set #2: The boundaries of the empirical parameters are defined from the expectations of the nuclear physics. We then have calculated the β equilibrium nuclear matter by using the meta-model and put them into the general relativistic equations for generating mass radius and tidal deformability. Besides, we defined constraints from nuclear physics (χ EFT and ISGMR) and the tidal deformabilities from the GW170817 event.

On the nuclear physic side, we used χ EFT predictions at near/below of the saturation density (n_{sat}) from Ref. [32] and the experimental value of M_c from the results of the ISGMR from the Refs. [33, 34]. Additionally, three types of tidal deformability probability distribution functions were considered: TD-IVC-2018, TD-De-2018 and TD-Coughlin-2019. TD-IVC-2018 is the result of the latest analysis from the LIGO-Virgo collaboration [26] while TD-De-2018 is an independent analysis proposed in Ref. [30] and TD-Coughlin-2019 is a combined analysis including the electro-magnetic and GRB counterpart observations from Ref. [31]. Finally, we showed how to generate posterior probabilities of each empirical parameters (L_{sym} , K_{sym} , Q_{sat} and Q_{sym}) and NS properties ($P(2n_{\text{sat}})$ and $R_{1.4}$) by taking advantage of the Bayesian framework.

NEUTRON STARS: PROPERTIES AND NUCLEAR EQUATION OF STATE PARAMETERS

5.1 An Overview

Taking advantages of the Bayesian framework, we analyze the impact of the constraints (C2)-(C4) (see Sec. 4.9.1 for details) to analyze each individual contribution coming from $\tilde{\Lambda}$, χ^{EFT} and ISGMR on the final posterior probability. Both joint and sole posterior probabilities will be shown. The influence of the prior set and two $p_{\tilde{\Lambda}}$ are also discussed. The uncertainty on probabilities are defined as the 68% confidence level around the centroid values of PDF.

In the present statistical analysis, we generate a large enough sample of 294 151 parameter sets for prior set #1 and 203 643 for prior set #2 before the filtering (see Table 4.2). For each set, the probabilities $p_{\tilde{\Lambda}}$, $p_{\chi^{\text{EFT}}}$ and p_{ISGMR} are calculated according to Eqs. (4.55), (4.57) and (4.61). The total likelihood probability is calculated from Eq. (4.54). The reduction from the multi-dimension PDF to the one- or two-parameter probabilities are obtained from the marginalization principle (see Eqs. (4.52) and (4.53) for details). We analyze the PDF for L_{sym} , K_{sym} , Q_{sat} , Q_{sym} , $R_{1.4}$, $P(2n_{\text{sat}})$ and the correlations between the parameters L_{sym} - K_{sym} and K_{sat} - Q_{sat} under the influence of each constraint associated to $p_{\tilde{\Lambda}}$ (TD-LVC-2018, TD-Le-2018 and TD-Coughlin-2019 as named in figures), $p_{\chi^{\text{EFT}}}$ (χ^{EFT} as named in figures) and p_{ISGMR} (GMR as named in figures). We also investigated the PDF for K_{sat} . However it is not shown here since K_{sat} has only a weak impact on $p_{\tilde{\Lambda}}$.

5.2 Probability Distributions for the Nuclear EoS Parameters

We first study posterior distributions for nuclear EoS parameters: L_{sym} , K_{sym} , Q_{sat} , Q_{sym} .

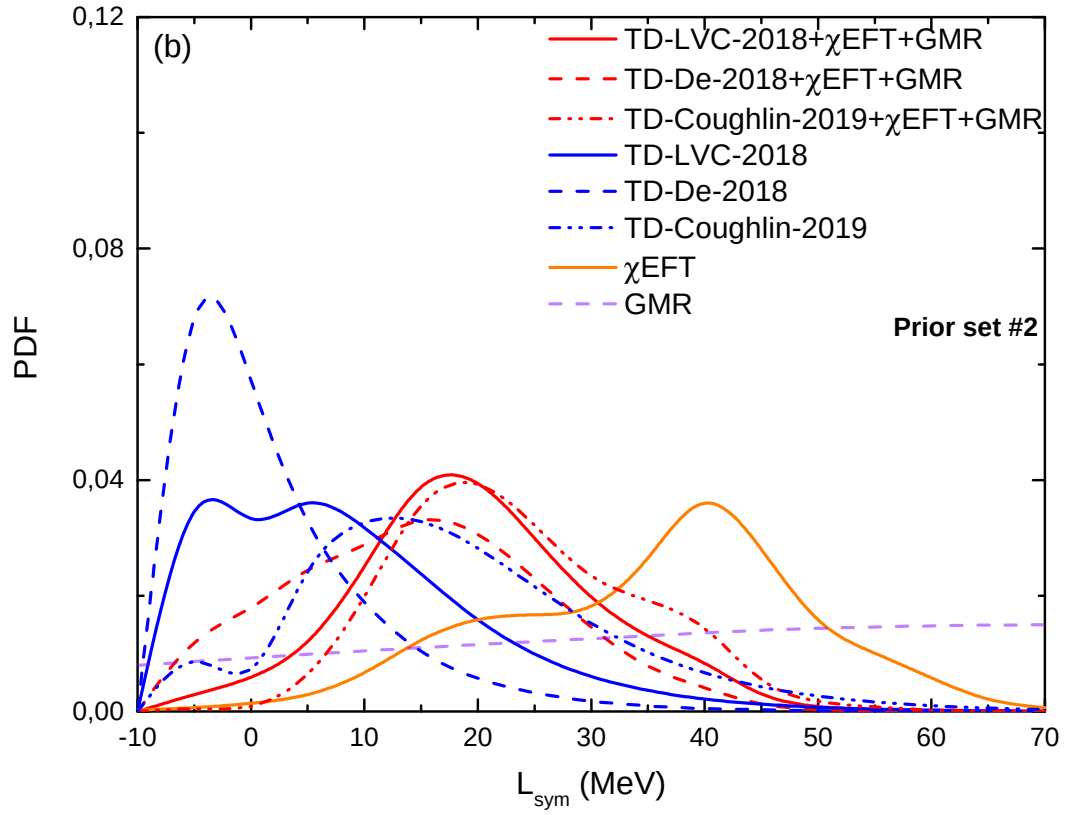
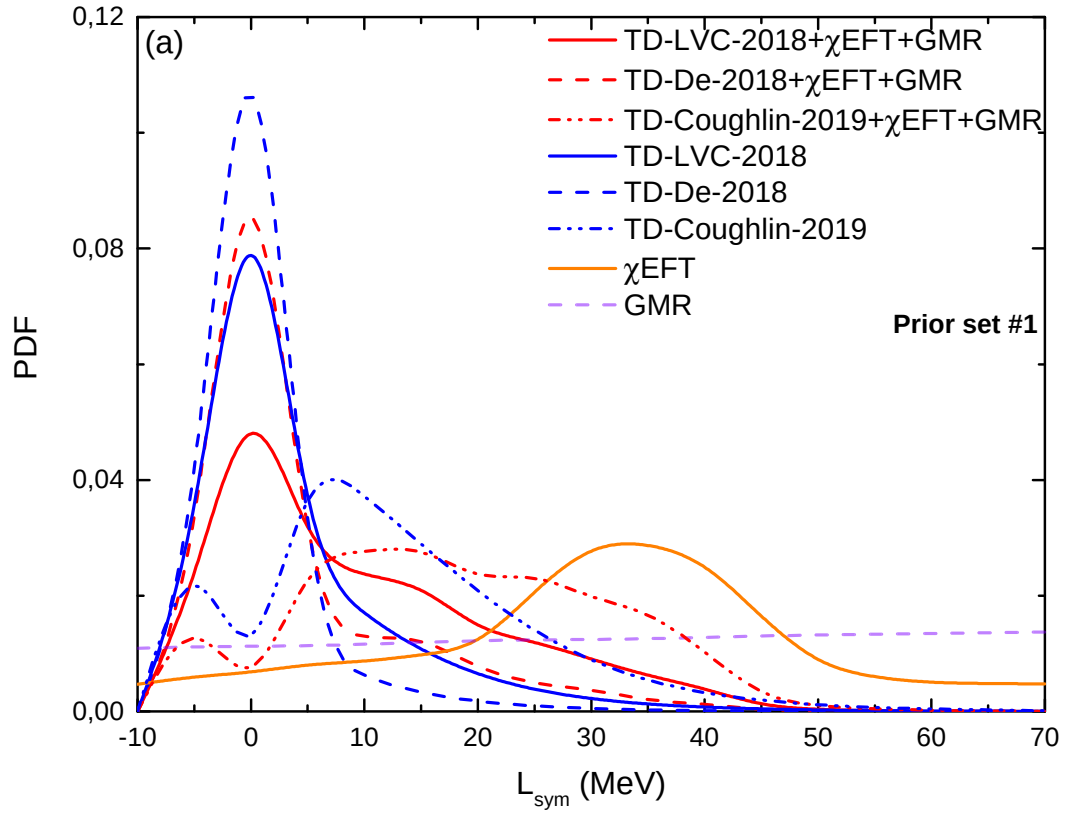


Figure 5.1 The generated PDFs of L_{sym} for the prior set #1 (a) and the prior set #2 (b)

5.2.1 The Slope of Symmetry Energy: L_{sym}

The empirical parameter L_{sym} is the slope of the symmetry energy at n_{sat} . In Fig. 5.1 the detailed contributions of the constraints (C2)-(C4) as well as of the role of the $p_{\tilde{\Lambda}}$ and of the prior scenario #1 (panel a) or #2 (panel b) is shown. Note the marked tension between the PDF associated to χ EFT and the $\tilde{\Lambda}$ one (TD-LVC-2018, TD-Le-2018, TD-Coughlin-2019). Being peaked at higher values for $\tilde{\Lambda}$, the TD-Coughlin-2019 PDF favors slightly larger L_{sym} values than the two others. The influence of the prior is weak, but interestingly, the prior set #1 produces more peaked posteriors than the prior set #2, which is inferred from analyses of nuclear physics models. This could be interpreted as a signal for the marked deviations from nuclear physics predictions: when the constraints from nuclear physics is relaxed (mainly the prior on K_{sym}) in the set #1, there is a group of EoS which are clearly favored by the GW tidal deformability and which are located well outside the domain for L_{sym} suggested by nuclear physics.

The GMR constraint has no effect on L_{sym} since the GMR mainly contributes to parameters related to symmetric nuclear matter. The χ EFT constraint gives values for $L_{\text{sym}} = 35.37^{+7.09}_{-10.10}/41.83^{+7.33}_{-15.82}$ MeV for the prior set #1/#2, while the tidal deformability favors low or even negative L_{sym} values. For instance, TD-LVC-2018 gives $L_{\text{sym}} = 0.00^{+5.00}_{-3.00}/-3.44^{+18.34}_{-2.94}$ MeV for the prior sets #1/#2. As expected, the prior set #2 allows some positive values for L_{sym} in the PDF shown in Fig. 5.1.

The joint probabilities naturally favor values for L_{sym} which are intermediate between the two extremes. The most probable value for TD-LVC-2018 (TD-De-2018 and TD-Coughlin-2019) is $L_{\text{sym}} = 0.00^{+12.00}_{-4.00}/16.58^{+11.28}_{-6.79}$ MeV ($L_{\text{sym}} = 0.00^{+2.01}_{-2.91}/15.47^{+11.24}_{-13.07}$ MeV and $L_{\text{sym}} = 17.44^{+15.23}_{-15.23}/16.48^{+14.73}_{-5.43}$ MeV) for the prior set #1/#2. The difference between the prior sets #1 (panel a) and #2 (panel b) reflects the choice for the prior distribution: the upper bound for K_{sym} is fixed to be 1500 MeV for the prior set #1 and only 300 MeV for the prior set #2 (see Table 4.2). The distribution of L_{sym} is thus impacted by the knowledge from the next order empirical parameter K_{sym} : The better defined K_{sym} , the more peaked L_{sym} . The correlation between L_{sym} and K_{sym} will be analysed in Sec. 5.4. Note that the influence of the unknown high order empirical parameters was originally stressed in Ref. [138].

Interestingly, the empirical parameter L_{sym} is investigated by a large number of experiments, see Ref. [139] and references therein. Confronting the predictions of various nuclear physics experiments, namely neutron skin thickness, heavy ion collisions, dipole polarizability, nuclear masses, giant dipole resonances and isobaric analog states, the values of L_{sym} vary between 30 and 70 MeV [30, 37, 139, 140]. It is however interesting to note that a few studies give for L_{sym} lower values, even negative ones, see Refs. [141, 142], from the charge radius of Sn and Pb isotopes using a droplet

model. A detailed analysis based on a few Skyrme and Gogny interactions advocates also for a low values for L_{sym} [143]. The measurement of the ^{208}Pb neutron skin thickness from the PREX collaboration (Lead Radius Experiment [144]) is expected to provide a model independent estimation of L_{sym} . The experiment has however not yet been very conclusive, with a measured neutron skin thickness $R_{\text{skin}}^{208} = 0.33^{+0.16}_{-0.18}$ fm points a lower limit for L_{sym} which is about 20 MeV if one includes the correlation for L_{sym} and R_{skin}^{208} (see Ref. [143] for details).

Anticipating the results of Sec. 5.3, there is a strong correlation between the marginalized probability distribution as function of L_{sym} and the one as function of $R_{1.4}$: a low value of L_{sym} coincides with a low radius $R_{1.4}$. Hence the peak at low L_{sym} observed for the tidal deformabilities TD-LVC-2018 and TD-De-2018 reflects that the $\tilde{\Lambda}$ PDF favors NS with small radii. Since the physical implications are clearer in terms of radii, we further discuss the implication of low radii (equivalently low L_{sym}) in Sec. 5.3.

5.2.2 The Curvature of the Symmetry Energy: K_{sym}

The empirical parameter K_{sym} encodes the curvature of the symmetry energy at n_{sat} . It is different from the parameter K_{τ} which is defined as the curvature of the binding energy for a fixed proton fraction and can be related to other nuclear EoS parameters as follows [126],

$$K_{\tau} \equiv K_{\text{sym}} - 6L_{\text{sym}} - Q_{\text{sat}}L_{\text{sym}}/K_{\text{sat}}. \quad (5.1)$$

The isospin dependence of the isoscalar giant monopole resonance (ISGMR) is a natural observable to determine the parameter K_{τ} [126]. $K_{\tau} = -550 \pm 100$ MeV has been extracted from the breathing mode of Sn isotopes (Refs. [145, 146]) and also from isospin diffusion observables in nuclear reactions (Refs. [147, 148]). If L_{sym} and Q_{sat} were well determined, Eq. (5.1) would provide an equivalence between K_{τ} and K_{sym} . However, the large uncertainties on L_{sym} and Q_{sat} induce a large error bar for K_{sym} , of the order of ± 600 MeV [37]. Besides, the statistical analysis of various theoretical model predict a value $K_{\text{sym}} = -100 \pm 100$ MeV [37]. This result is also in agreement with Ref. [115], which is GW analysis done by using TE EoSs. On the other hand, there is an experimental determination of K_{sym} by using latest ISGMR values of ^{90}Zr , ^{116}Sn and ^{208}Pb nuclei from Skyrme EDFs: $K_{\text{sym}} = -120 \pm 40$ MeV from Ref. [149]. The smaller error bar than the statistical analysis reveals the presence of correlations between L_{sym} , Q_{sat} and K_{sym} which does not vary independently from each other.

In our analysis, we explore two priors for K_{sym} , one which is pushed until the likelihood probability is quenched (prior set #1), and one which is compatible

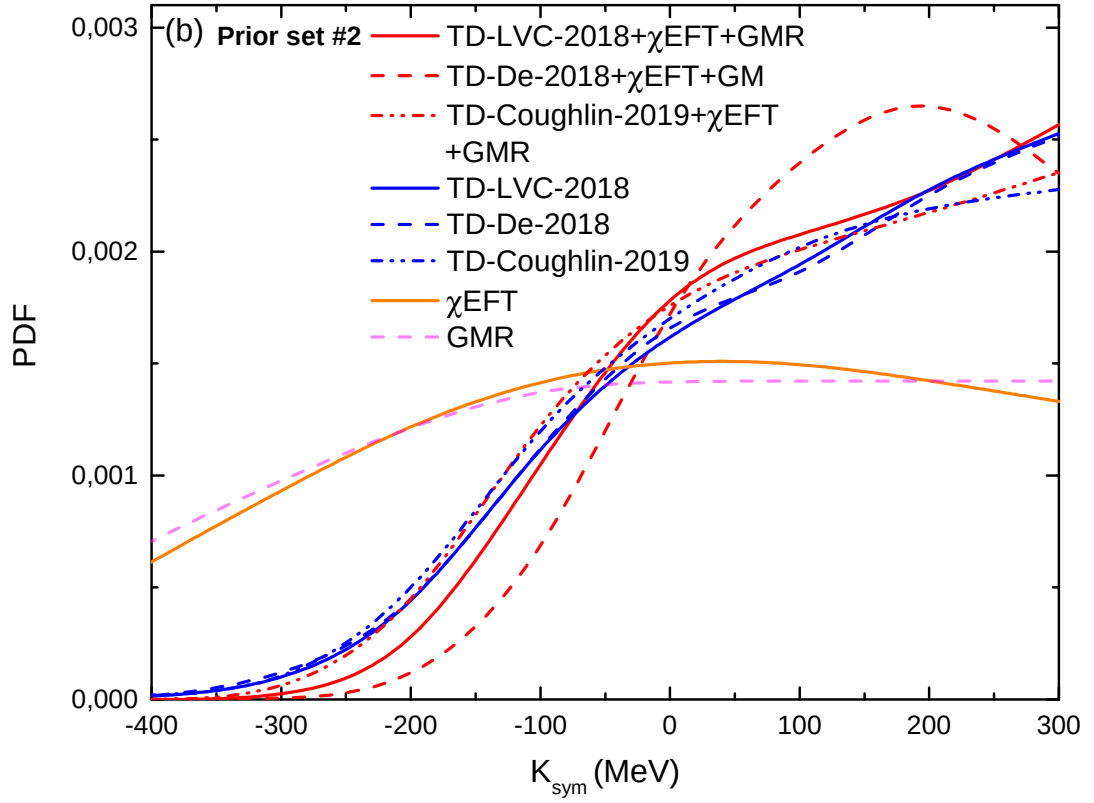
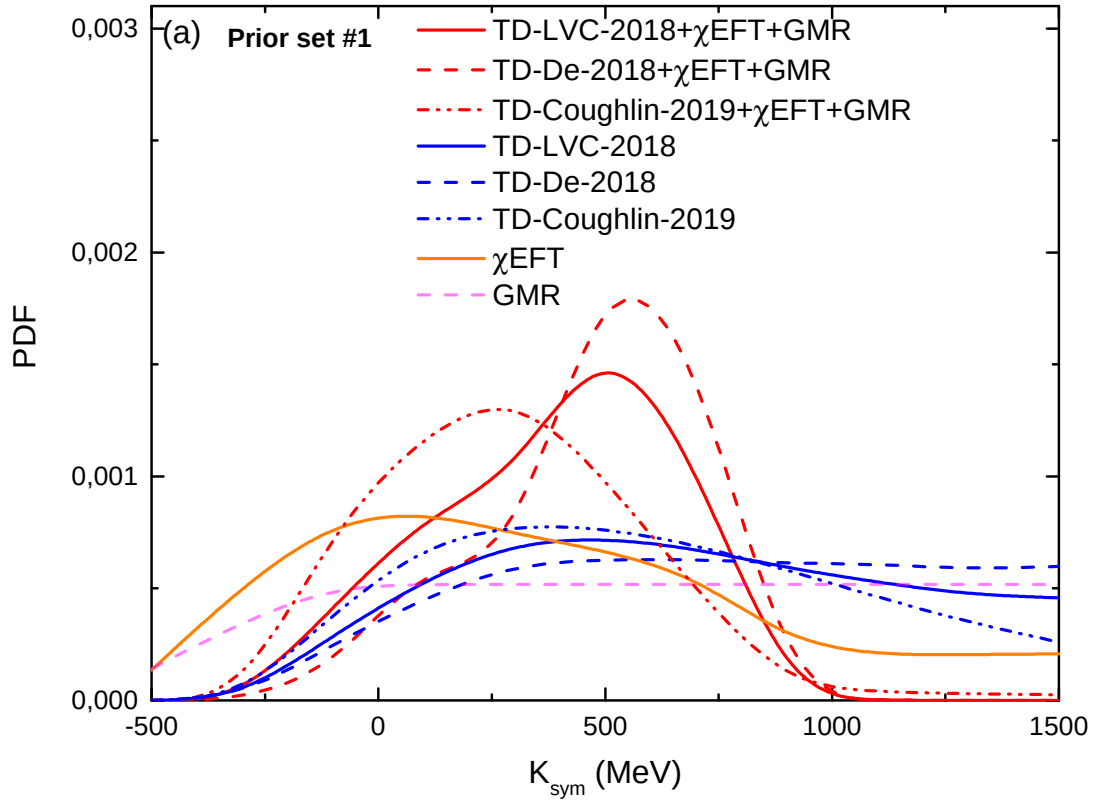


Figure 5.2 The generated PDFs of K_{sym} for the prior set #1 (a) and #2 (b)

with the expectation $K_{\text{sym}} = -100 \pm 100$ MeV (prior set #2). In Fig. 5.2, the posterior PDFs for K_{sym} are displayed for both prior sets. The posteriors are qualitatively similar between the prior sets #1 and #2. From χ EFT, we obtain $K_{\text{sym}} = 13.71^{+595.94}_{-265.02}/12.58^{+287.42}_{-410.00}$ MeV for the prior set #1/#2. The tidal deformability however favors positive values since TD-LVC-2018 (TD-De-2018 and TD-Coughlin-2019) predicts $K_{\text{sym}} = 376.44^{+1123.46}_{-400}$ MeV ($K_{\text{sym}} = 389.65^{+1110.45}_{-400}$ and $K_{\text{sym}} = 273.82^{+888.16}_{-330.93}$ MeV) for the prior set #1. TD-Coughlin-2019 favors values for K_{sym} slightly below the distributions produced by TD-LVC-2018 and TD-De-2018. This can be understood from the L_{sym} - K_{sym} anti-correlation originating in the causality condition. Although we cannot define centroid values of K_{sym} since the prior set #2 limits the posteriors to $K_{\text{sym}} = 300$ MeV, shifting the prior set #1 to #2 adds 100 MeV to the minimum values of K_{sym} . There is also a difference between the expectations from χ EFT and from the tidal deformability, while at variance with L_{sym} , the differences are here less marked. The impact of the ISGMR is also pretty small.

Finally, the joint probabilities shown in Fig. 5.2 give $K_{\text{sym}} = 438.57^{+210.12}_{-210.12}$ MeV ($K_{\text{sym}} = 561.20^{+150.23}_{-150.23}$ MeV and $K_{\text{sym}} = 261.00^{+237.14}_{-237.14}$ MeV) for TD-LVC-2018 (TD-De-2018 and TD-Coughlin-2019). Considering the $-2\sigma_{\text{min}}$ value for each centroid, one can define the lower limit for K_{sym} : $K_{\text{sym}} \geq 18.33$ MeV for TD-LVC-2018, $K_{\text{sym}} \geq 260.74$ MeV for TD-De-2018 and $K_{\text{sym}} \geq -213.28$ MeV for TD-Coughlin-2019. It should be noted that several analysis have been done on the bounds of K_{sym} , providing $K_{\text{sym}} \geq -500$ MeV to $K_{\text{sym}} \geq -250$ MeV depending on considered models [150–153]. Besides, an interesting work about the lower limit of K_{sym} is the Unitary Gas (UG) limit for the NM, which is in a good agreement with our predictions [154]. Since the ground state energy per particle in the UG is proportional to the Fermi energy, one can describe a forbidden zone for energy per particle of EoS in terms of the Fermi energy for neutron matter. In Ref. [154], a suitable conjecture imposed from the UG limit is shown: $E_{\text{NM}} \geq E_{\text{UG}} = E_{\text{UG}}^0 n_0^{2/3}$. Eventually it leads to $K_{\text{sym}} \geq -2E_{\text{UG}}^0 - K_{\text{sat}}$ where $E_{\text{UG}}^0 = 12.6$ MeV is the Fermi energy of neutrons including Bertsch parameter ξ_0 . Using the average value of $K_{\text{sat}} = 230 \pm 20$ MeV (see Ref. [37] for a complete analysis about the parameter K_{sat}), a minimum limit for K_{sym} can be obtained: $K_{\text{sym}} \geq -255.2 \pm 20$ MeV. However, contrary to the UG, the NM includes effective-range effects and interactions in higher partial waves especially for densities $n \geq n_{\text{sat}}$. Therefore, it is expected that the lower limit of K_{sym} should be higher then the one obtained from the UG.

5.2.3 The Skewness Parameter for Symmetric Matter: Q_{sat}

The skewness parameter Q_{sat} is the lowest order empirical parameter in SM which is almost unconstrained. While the incompressibility modulus K_{sat} is well defined, the

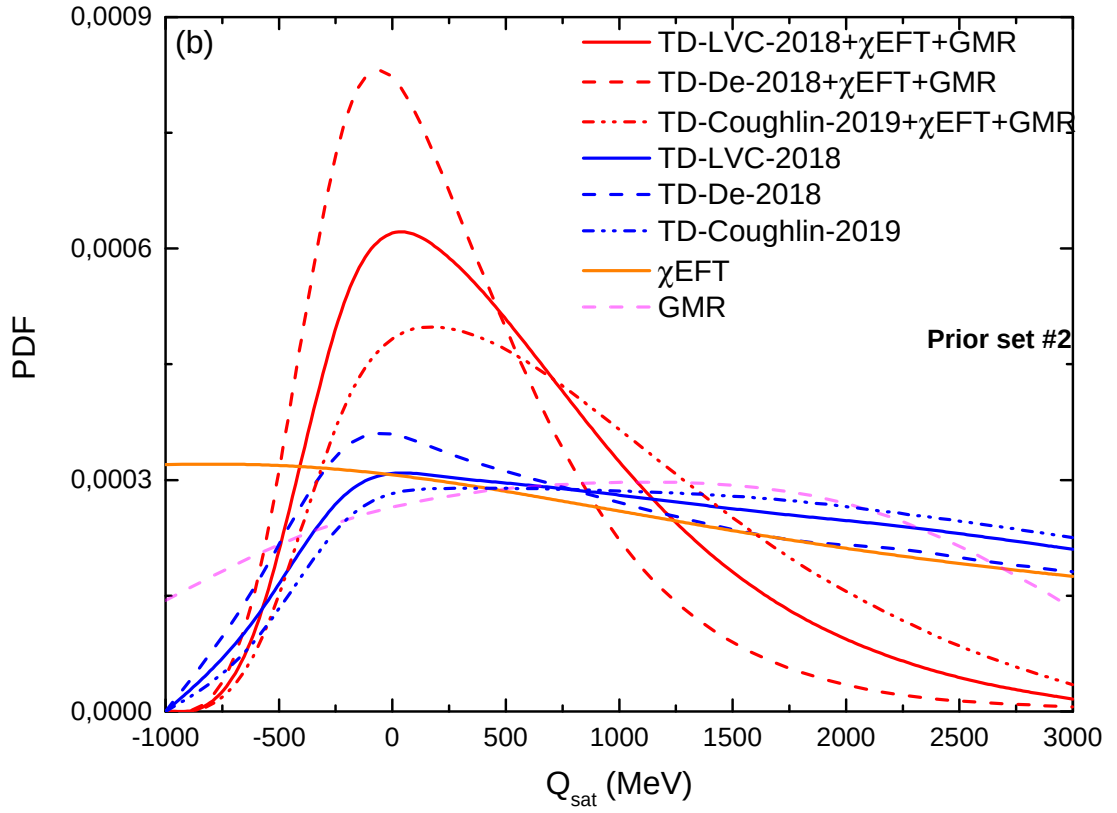
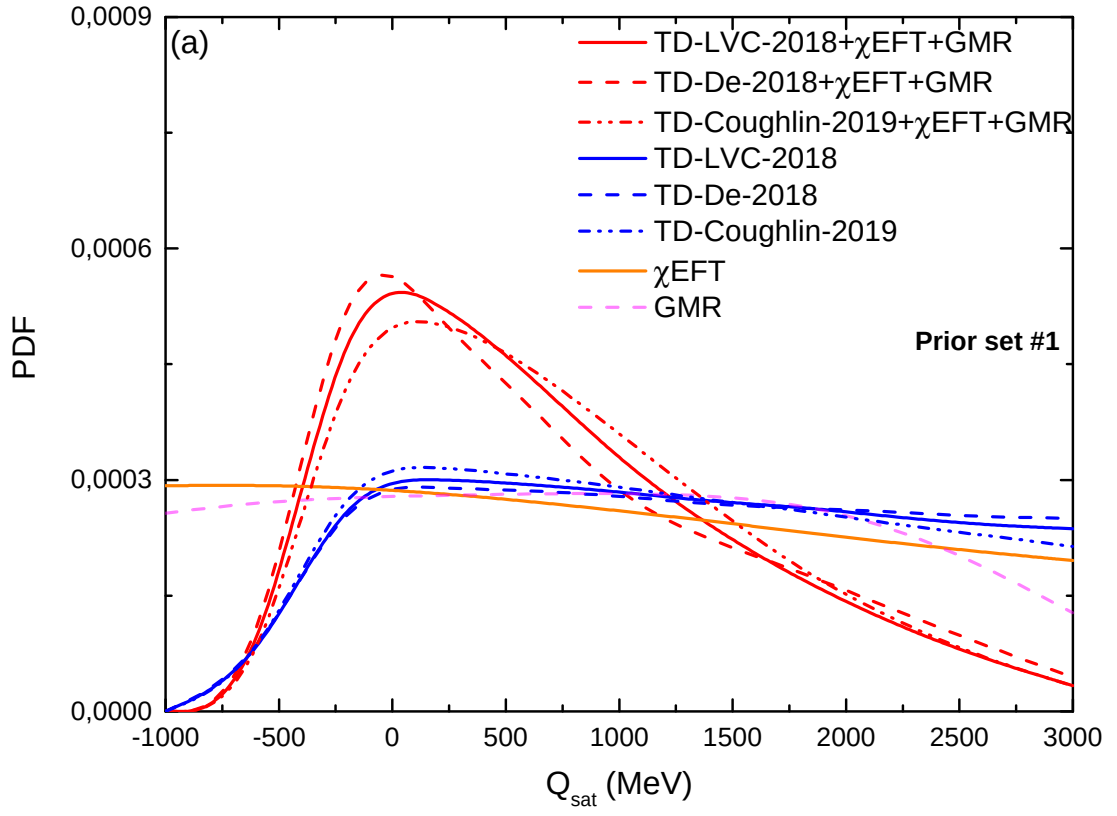


Figure 5.3 The generated PDFs of Q_{sat} for the prior set #1 (a) and #2 (b)

density dependence of the incompressibility is poorly known and there are very scarce experimental analysis to determine its value. An analysis based on charge and mass radii of the Sn isotopes concluded that either $Q_{\text{sat}} \approx 30$ MeV or $L_{\text{sym}} \approx 0$ MeV [142]. Another analysis based on the Skyrme functionals which are fitted according to the breathing modes concluded that $Q_{\text{sat}} \approx 500$ MeV [155]. A systematic analysis also suggests $Q_{\text{sat}} = 300 \pm 400$ MeV based on a large number of theoretical models of the literature [37].

There are also other analysis based on various models from the RMF and SHF frameworks in which the tidal deformability of GW170817 constrains the parameter M_0 of the nuclear EoS defined as [115, 117],

$$M_0 = M(n_{\text{sat}}) = 3n_{\text{sat}} \left. \frac{dK(n_0)}{dn_0} \right|_{n_0=n_{\text{sat}}} . \quad (5.2)$$

The following predictions were obtained for M_0 : $2254 \leq M_0 \leq 3631$ MeV or $1926 \leq M_0 \leq 3768$ MeV depending on L_{sym} [117] and $1526 \leq M_0 \leq 4971$ MeV [115].

Using the relation $M_0 = 12K_{\text{sat}} + Q_{\text{sat}}$ (see Ref. [156]), one can make a prediction for Q_{sat} by considering adequate K_{sat} value. Considering $K_{\text{sat}} = 230 \pm 20$ MeV from Ref. [37], then $-800 \leq Q_{\text{sat}} \leq 1100$ MeV for Ref. [117] and $-1200 \leq Q_{\text{sat}} \leq 2100$ MeV for Ref. [115].

In Fig. 5.3, the posterior PDFs of Q_{sat} are presented. It is clear that χEFT does not constrain Q_{sat} . This is because Q_{sat} influences the EoS at densities well above saturation density, while the data from χEFT are relevant until $n_0 = 0.2 \text{ fm}^{-3}$. The empirical parameter Q_{sat} is however better constrained by both the tidal deformability from GW170817 and the ISGMR while the predictions from prior set #1 and #2 are very similar. Despite that all posteriors of tidal deformability considering TD-LVC-2018, TD-De-2018 or TD-Coughlin-2019 independently agree on the lower limit of Q_{sat} ($Q_{\text{sat}}^{\text{min}} \approx -500$ MeV), the higher boundary of Q_{sat} requires by applying both the tidal deformability and the ISGMR constraints. The results from the joint posteriors are $Q_{\text{sat}} = -180_{-175}^{+1222} / -162_{-175}^{+935}$ MeV ($Q_{\text{sat}} = -220_{-150}^{+1130} / -214_{-153}^{+652}$ MeV and $Q_{\text{sat}} = 93_{-250}^{+1365} / 200_{-445}^{+1107}$ MeV) for TD-LVC-2018 (TD-De-2018 and TD-Coughlin-2019) for the prior set #1/#2, respectively.

Furthermore, we also study the impact of switching off the ISGMR constraint for the prior set #1 on the posterior probability in order to see its global effect on the joint posteriors, see Fig. 5.4. The new the joint posteriors are $Q_{\text{sat}} = -134_{-250}^{+1757}$ MeV ($Q_{\text{sat}} = -189_{-200}^{+1800}$ MeV and $Q_{\text{sat}} = -130_{-250}^{+2000}$ MeV) for TD-LVC-2018 (TD-De-2018 and TD-Coughlin-2019). Removing the ISGMR constraints increases the uncertainty

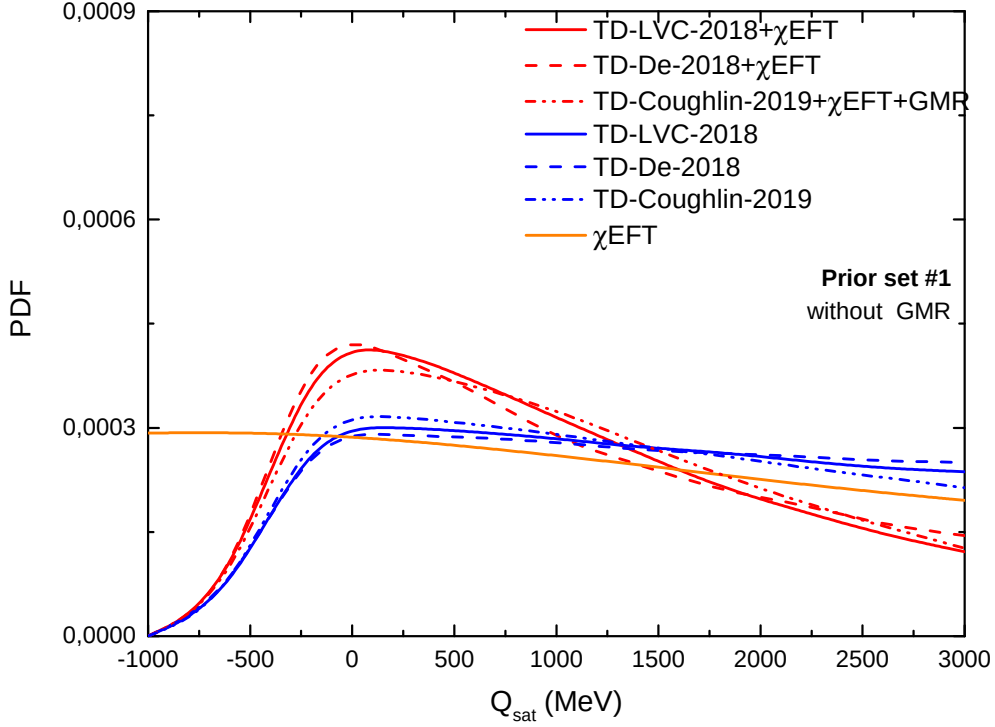


Figure 5.4 Same as Fig. 5.3 for the prior set #1 without ISGMR

on the joint posteriors for Q_{sat} by about 500 MeV. This shows that M_c (see Sec. 4.9.3 for details) is an important constraint for better defining the value of Q_{sat} . Furthermore, a reduction of the uncertainty on M_c , by a systematical comparison of the meta-model predictions in finite nuclei for instance, would imply a more precise estimation for the empirical parameter Q_{sat} .

5.2.4 The Skewness Parameter for Neutron Matter: Q_{sym}

The nuclear EoS parameter Q_{sym} controls the skewness of the symmetry energy at n_{sat} . An analysis based on the various theoretical models (Skyrme Hartree Fock, Relativistic Hartree Fock, RMF and χ EFT) suggests $Q_{\text{sym}} = 0 \pm 400$ MeV but still its value runs over a large range from models to models, e.g. $-2000 \leq Q_{\text{sym}} \leq 2000$ MeV [37]. Since Q_{sym} contributes to the EoS at supra-saturation densities, it is quite difficult to estimate the value of this empirical parameter from low-density χ EFT or from terrestrial experiments in finite nuclei like the ISGMR. It furthermore requires systems which probe asymmetric nuclear matter. It is therefore completely unknown from nuclear physics traditional approach and one could easily understand that χ EFT and ISGMR constraints are ineffective for constraining Q_{sym} , as shown in Fig 5.5. The most effective constraint is provided by the tidal deformability, but it is interesting to remark that even if χ EFT and ISGMR do not provide constraints taken

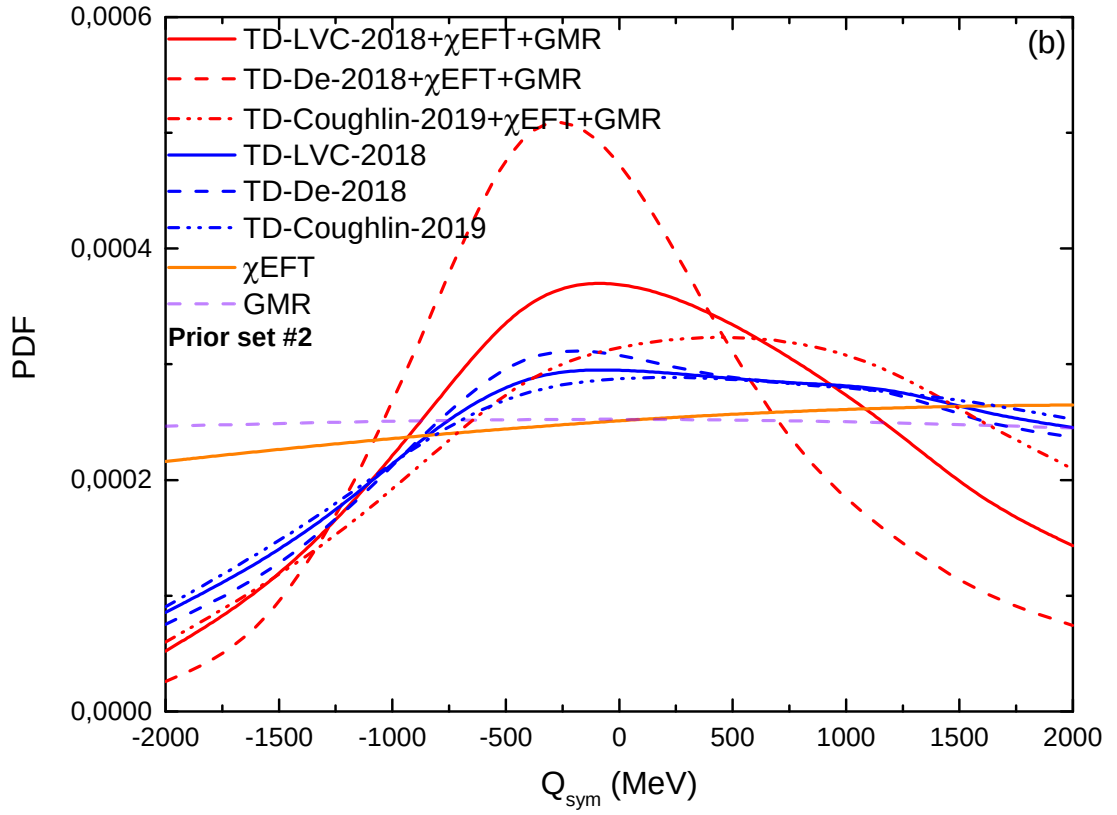
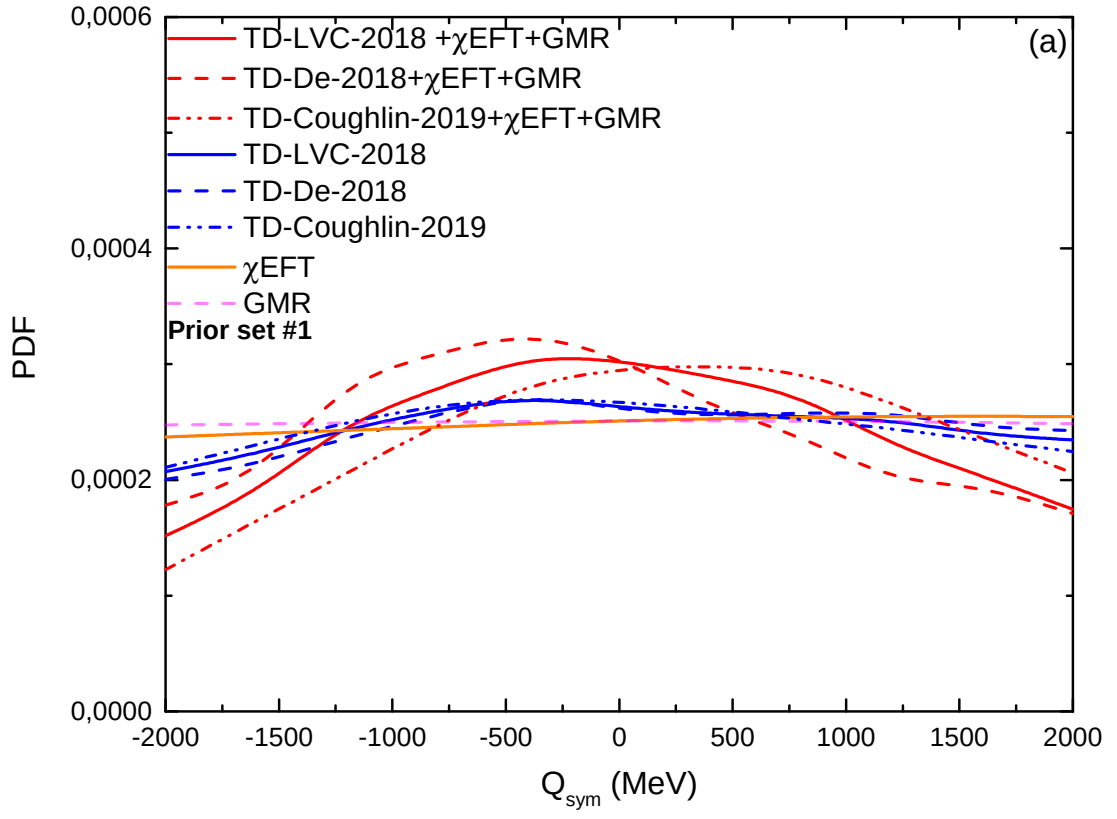


Figure 5.5 The generated PDFs of Q_{sym} for the prior set #1 (a) and #2 (b)

individually, the joint posterior, including tidal deformability, χ EFT and ISGMR, is narrower than the probability distribution considering $\tilde{\Lambda}$ alone. The joint posteriors from TD-LVC-2018 (TD-De-2018 and TD-Coughlin-2019) favor the following values: $Q_{\text{sym}} = -270_{-1126}^{+1690} / -169_{-748}^{+1376}$ MeV ($Q_{\text{sym}} = -677_{-597}^{+1159} / -376_{-477}^{+835}$ MeV and $Q_{\text{sym}} = 218_{-1576}^{+1942} / 276_{-1242}^{+1815}$ MeV) for the prior set #1/#2. It shall also be noted that there is a marked correlation between K_{sym} and Q_{sym} : the prior set #2, considering a tighter prior for K_{sym} , (compared to the prior set #1) also predicts a narrower peak for Q_{sym} . In conclusion, we point out that a more accurate PDF for $\tilde{\Lambda}$, a better constrain for Q_{sym} .

5.3 Posteriors for Neutron Star Observables: Radius and Pressure

Let us discuss the impact of the posteriors on the NS properties. In the present section, we discuss the impact of the constraints on the posterior distribution for the NS radius at $1.4M_{\odot}$: $R_{1.4}$, and the pressure at $2n_{\text{sat}}$: $P(2n_{\text{sat}})$.

5.3.1 The Neutron Star Radius at $1.4M_{\odot}$: $R_{1.4}$

As discussed in Sec. 4.2, X-ray observations of NS such as thermal emissions or X-ray bursts, advocate for the following limits of NS radii: $7.9 \leq R_{1.4} \leq 12.66$ km [6, 98, 99, 102, 103]. Moreover, GW analysis based on various models concluded to $11.80 \leq R_{1.4} \leq 12.80$ km in Ref. [118], $12.00 \leq R_{1.4} \leq 13.70$ km in Refs. [111, 112], and $11 \leq R_{1.4} \leq 13$ km considering $100 \leq \tilde{\Lambda} \leq 600$ in Ref. [114]. While being consistent among them, these predictions are slightly different, reflecting the small model dependence in the theoretical models employed.

We show in Fig. 5.6 the posteriors PDFs for the NS radius $R_{1.4}$ for the different individual constraints and for the joint one. The predictions from TD-LVC-2018 and TD-De-2018 are $R_{1.4} = 10.65_{-0.26}^{+2.1} / 10.51_{-0.17}^{+1.29}$ km for the prior set #1/#2 at variance with the prediction from TD-Coughlin-2019 $R_{1.4} = 13.13_{-0.51}^{+0.51}$ km, which are consistent with the predictions from nuclear physics (χ EFT): $R_{1.4} = 12.99_{-1.21}^{+0.76} / 12.72_{-0.61}^{+0.82}$ km for the prior set #1/#2. If the $\tilde{\Lambda}$ distribution suggested by TD-LVC-2018 and TD-De-2018 is correct, there is a difference of about 1.5 km for the most probable radii compared to the prediction from χ EFT. This difference is larger than the standard deviation for each PDF, indicating a possible source of tension, as also observed for the PDF of L_{sym} . Finally, the joint probabilities shown in Fig. 5.6 give $R_{1.4} = 11.00_{-0.25}^{+1.30} / 10.98_{-0.25}^{+1.90}$ or $R_{1.4} = 12.00_{-1.25}^{+0.30} / 10.98_{-0.25}^{+1.90}$ km ($R_{1.4} = 11.00_{-0.25}^{+1.25} / 10.99_{-0.25}^{+1.70}$ or $R_{1.4} = 12.00_{-1.25}^{+0.25} / 10.99_{-0.25}^{+1.70}$ km and $R_{1.4} = 12.91_{-0.43}^{+0.43} / 12.50_{-0.27}^{+0.54}$ km) for TD-LVC-2018 (TD-De-2018 and TD-Coughlin-2019) for

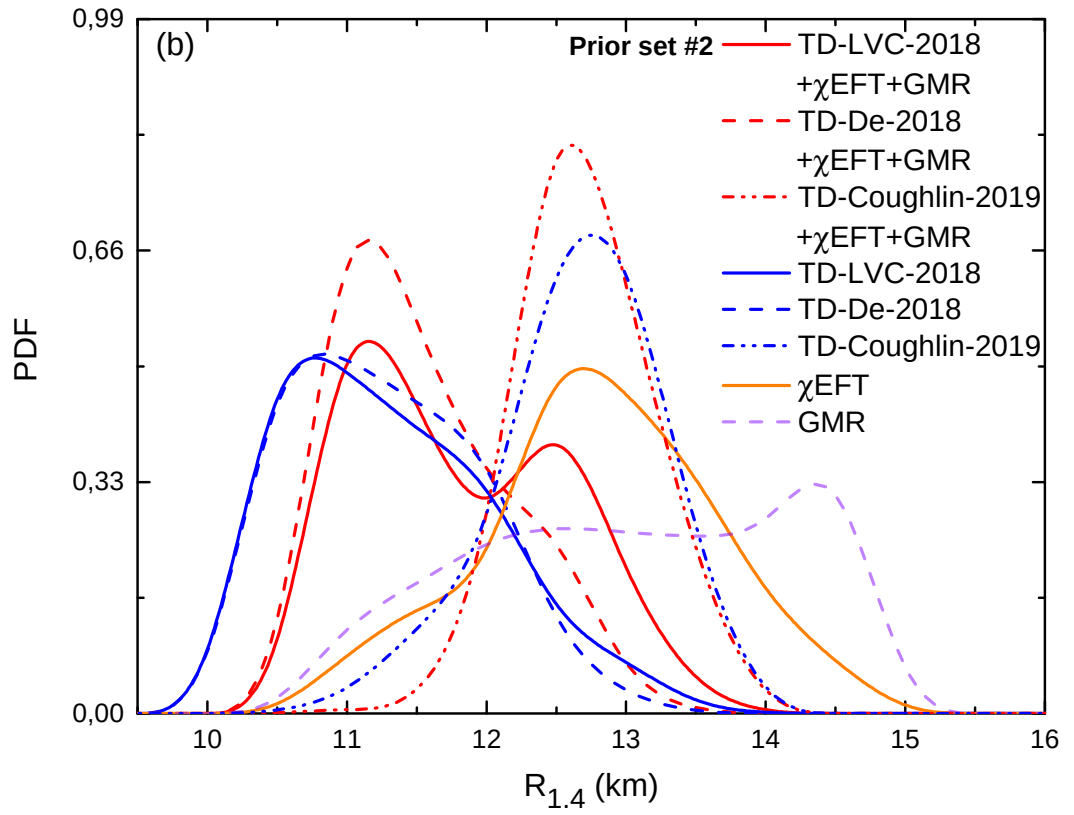
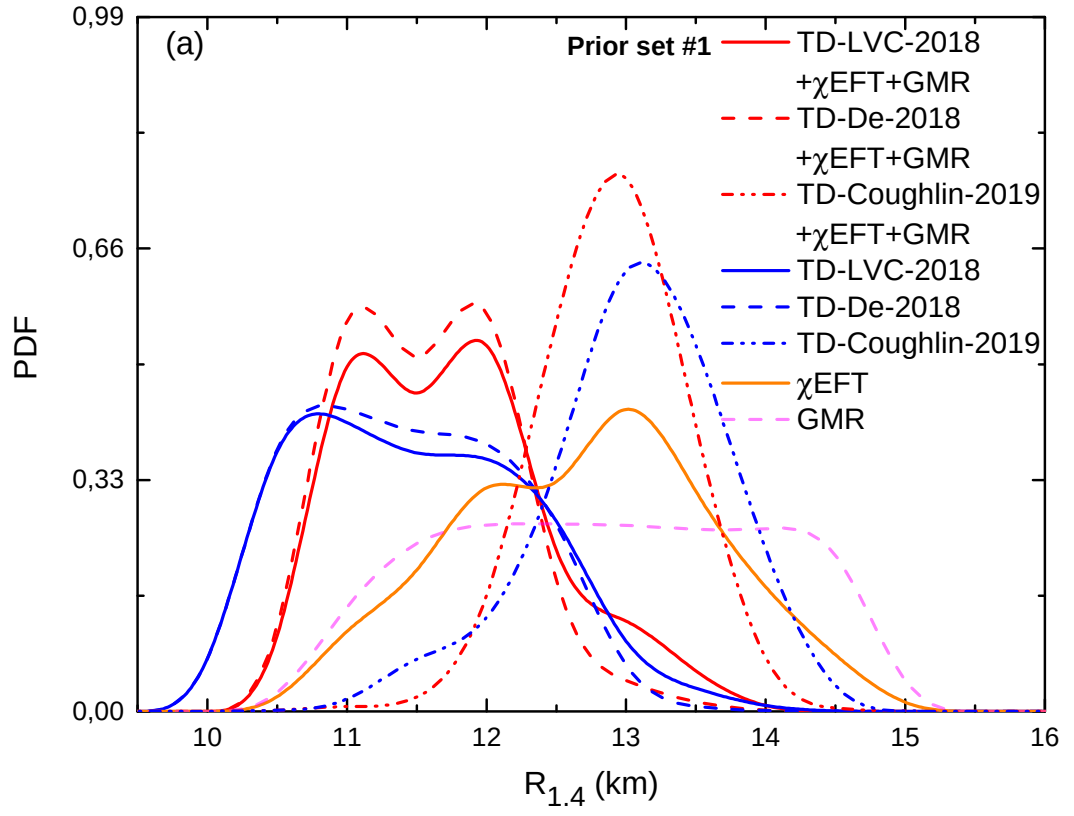


Figure 5.6 The generated PDFs of NS radius $R_{1.4}$ for the prior set #1 (a) and #2 (b)

the prior set #1/#2. Interestingly, the joint posteriors suggested by TD-LVC-2018 and TD-De-2018 predicts a double peak, where the first one is around 11 km and the second one is around 12 km for the prior set #1.

Our prediction for $R_{1.4}$ favored by GW170817 only (TD-LVC-2018 and TD-De-2018 but not TD-Coughlin-2019) is very similar to the one recently performed in Ref. [124], where $R_{1.4} \approx 11 \pm 1$ km is obtained from the analysis of the GW waveforms and the constraint from the maximum mass. This is not entirely surprising: even if the analysis is different from ours, namely relaying on the bare data of Ref. [124] and based on the post-processed analysis in terms of $\tilde{\Lambda}$ in our case, the physics issued from GW is the same. A low value for the radius $R_{1.4} \approx 11$ km is marginal with nuclear physics (represented here by the χ EFT and GMR constraints). This results of low value for the radius suggest that the low peak value for $\tilde{\Lambda} \approx 200$ needs a softening of the EoS that nuclear degrees of freedom could not produce for the typical masses estimated from GW170817, which are around $1.3 - 1.5M_{\odot}$ (coinciding to central densities of about $2 - 3\rho_{\text{sat}}$). This softening could be obtained by the onset of new degrees of freedom, such as pion or kaon condensation, hyperonization of matter or a first order phase transition to quark matter. The requirement to reach about $2M_{\odot}$ also limits the softening, which could be obtained assuming a transition to quark matter [18].

5.3.2 The Pressure at $2n_{\text{sat}}$: $P(2n_{\text{sat}})$

It was recently proposed to analyze the constraint from the tidal deformability from GW170817 in terms of the pressure at $2n_{\text{sat}}$ [157]. An analysis done by Ligo-Virgo collaborations [157] obtained (with 90% confidence interval) a pressure $P(2n_{\text{sat}}) = 21.80^{+15.76}_{-10.55}$ MeV fm $^{-3}$ where the error bars represent 90% confidence level (corresponding to $P(2n_{\text{sat}}) = 21.80^{+9.58}_{-6.41}$ MeV fm $^{-3}$ for 65% confidence level). Another analysis based on χ EFT [114] concluded that $15 \leq P(2n_{\text{sat}}) \leq 25$ MeV fm $^{-3}$ considering $100 \leq \tilde{\Lambda} \leq 600$.

We thus further extend this approach by also imposing nuclear physics constraints on top of the tidal deformability, in the same spirit of the previous plots (Fig.5.7). We have also added $P(2n_{\text{sat}})$ from Ref. [157] for comparison. The constraints from χ EFT and ISGMR generate a rather flat distribution between the boundaries with small and marginal peaks. The tidal deformability imposes slightly stronger constraints, with $P(2n_{\text{sat}}) \geq 15$ MeV for the prior set #1 and #2. It is however interesting to note that here also, the joint posteriors predicts a peak narrower when including all three constraints: $P(2n_{\text{sat}}) = 24.61^{+24.42}_{-5.00}/26.02^{+13.58}_{-5.00}$ MeV fm $^{-3}$ ($P(2n_{\text{sat}}) = 23.69^{+27.95}_{-5.00}/25.00^{+7.82}_{-5.21}$ MeV fm $^{-3}$ and $P(2n_{\text{sat}}) = 25.00^{+19.91}_{-5.00}/30.00^{+18.29}_{-6.69}$ MeV fm $^{-3}$) for TD-LVC-2018 (TD-De-2018 and TD-Coughlin-2019) for the prior set

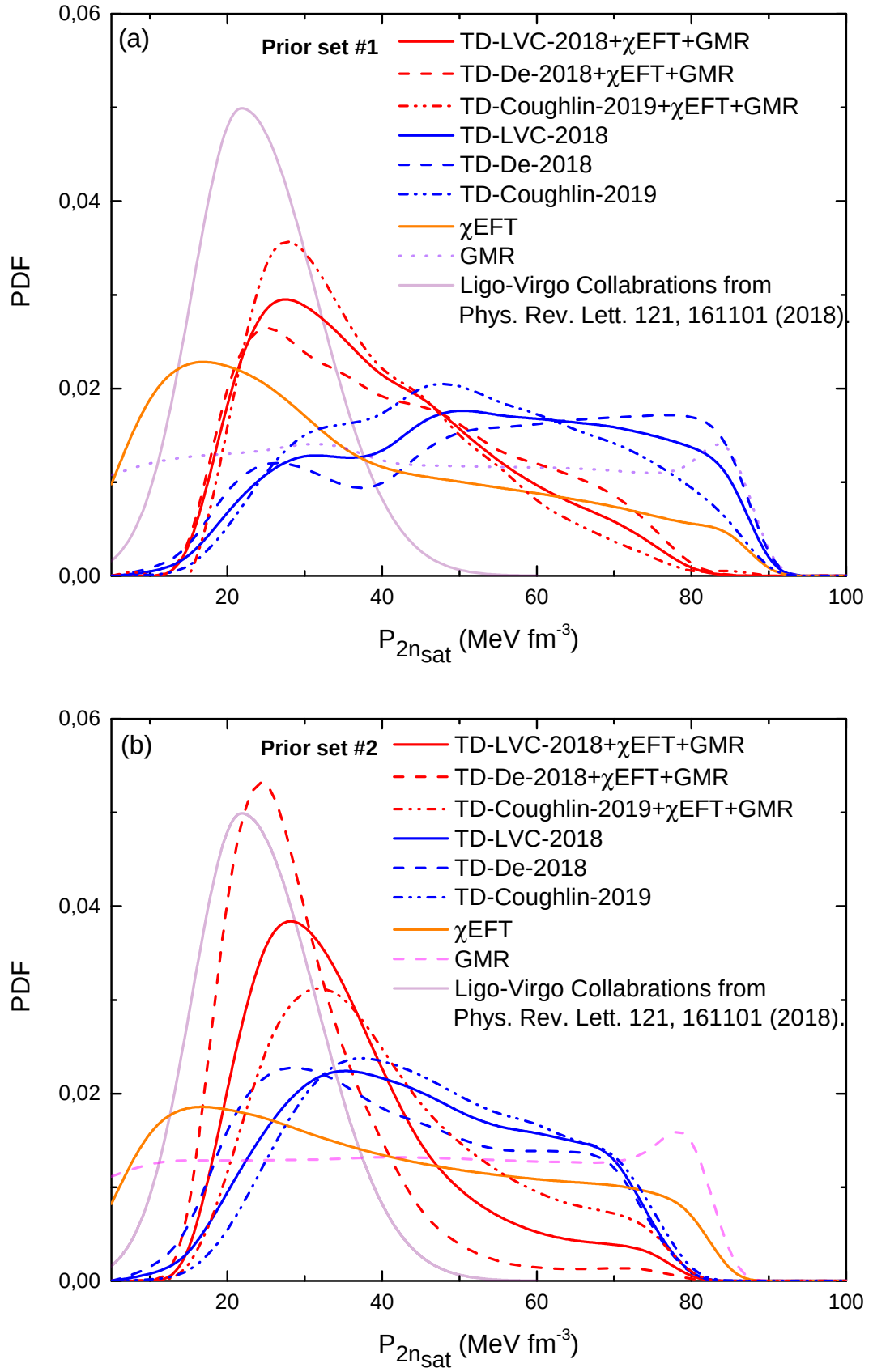


Figure 5.7 The generated PDFs of the pressure at $2n_{\text{sat}}$ for the prior set #1 (a) and #2 (b)

#1/#2. Although the centroid value of each tidal deformabilities are quite similar between the priors, the prior set #2 gives less uncertainty in the TD-LVC-2018 and TD-De-2018 cases. All in all, we conclude that the limits of the pressure at $2n_{\text{sat}}$ is: $19 \leq P(2n_{\text{sat}}) \leq 50 \text{ MeV fm}^{-3}$. Besides, considering the prior set #2 which has a tighter bound for K_{sym} , our prediction is in good agreement with the one proposed from Ligo-Virgo (Ref. [157]). The smaller dispersion is shown to come from the ISGMR, χEFT and tidal deformability considered all together. There is however no inclusion of quark phase transition in the present analysis, which is expected to increase the width of the prediction [108, 109].

5.4 Analysis of the Correlations Among the Nuclear Empirical Parameters

It is interesting to study the correlations among empirical parameters since they could sometimes reveal physical and universal correlations, or spurious correlations generated by the reduced number of free parameters. Therefore, the correlations $E_{\text{sym}} - L_{\text{sym}}$, $L_{\text{sym}} - K_{\text{sym}}$ and $K_{\text{sat}} - Q_{\text{sat}}$ are widely discussed [33, 34, 150–153, 158–161]. For instance, the correlation between K_{sat} and Q_{sat} typically found for Skyrme and Gogny interactions, is related to the presence of a single density dependent term in the nuclear force [33, 34]. Hence, a recent analysis of several of these correlations can be found in Ref. [138]. In the present section we provide an analysis on $L_{\text{sym}} - K_{\text{sym}}$ and $K_{\text{sat}} - Q_{\text{sat}}$ correlations under the influence of each constraint associated to $p_{\tilde{\Lambda}}$, $p_{\chi\text{EFT}}$ and p_{ISGMR} . The correlation between E_{sym} and L_{sym} is not shown here since we used a fixed E_{sym} value for prior sets #1/#2 (see Table 4.1 for details).

5.4.1 The Correlation between L_{sym} and K_{sym}

We first explore the correlation between L_{sym} and K_{sym} (see Figs. 5.8), which was also explored in Refs. [150–153, 158–161]. We remind that the influence of the prior sets on the PDF related to L_{sym} (Fig. 5.1), was suggesting the presence of a correlation between L_{sym} and K_{sym} . Here also we find a marked difference between the $L_{\text{sym}} - K_{\text{sym}}$ domain favored by the GW constrain (low L_{sym} values) and the one favored by the χEFT one (high L_{sym} values). The lower bounds in L_{sym} and K_{sym} are imposed by the stability and $M_{\text{max}}^{\text{obs}}$ constraints, while the upper bounds are fixed by the causality one. Note that the $L_{\text{sym}} - K_{\text{sym}}$ domain favored by the TD-De-2018 $\tilde{\Lambda}$ -PDF is a bit smaller than the one favored by TD-LVC-2018. Moreover, the prior set #2 exploring a smaller parameter space than the prior set #1 (see Tab. 4.2), the correlation domain is smaller for prior set #2 compared to #1. Despite this main difference, there is still a small but noticeable impact of the prior set.

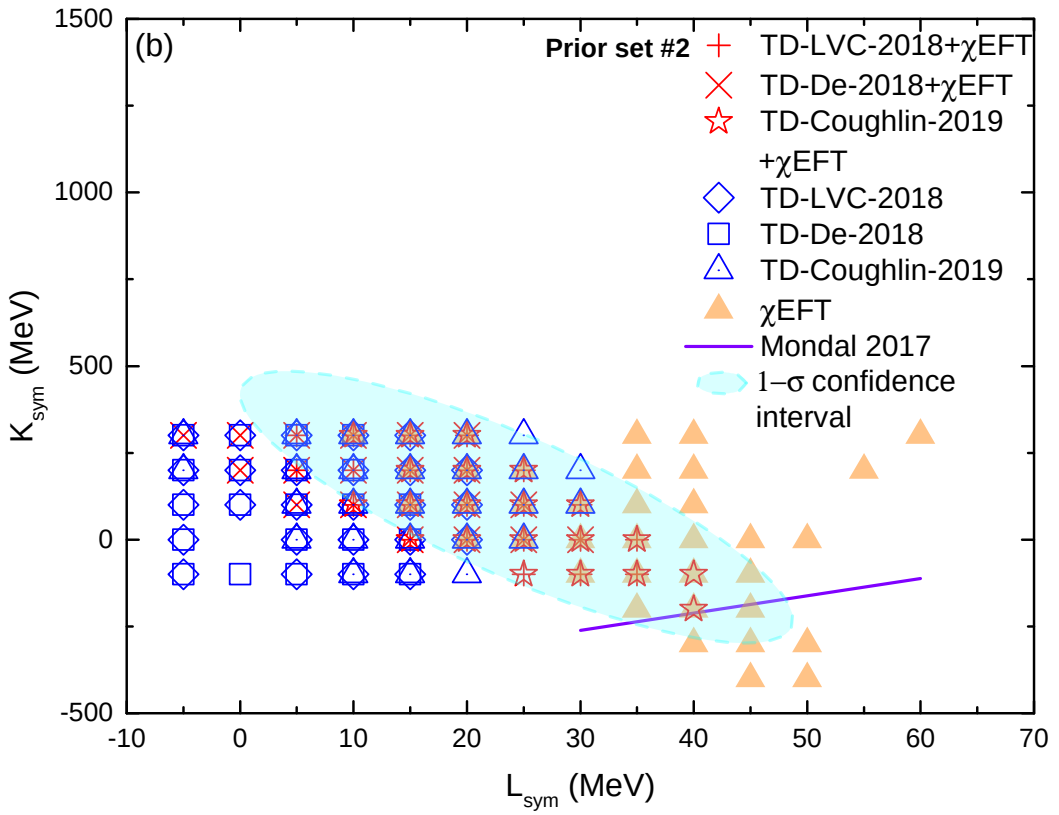
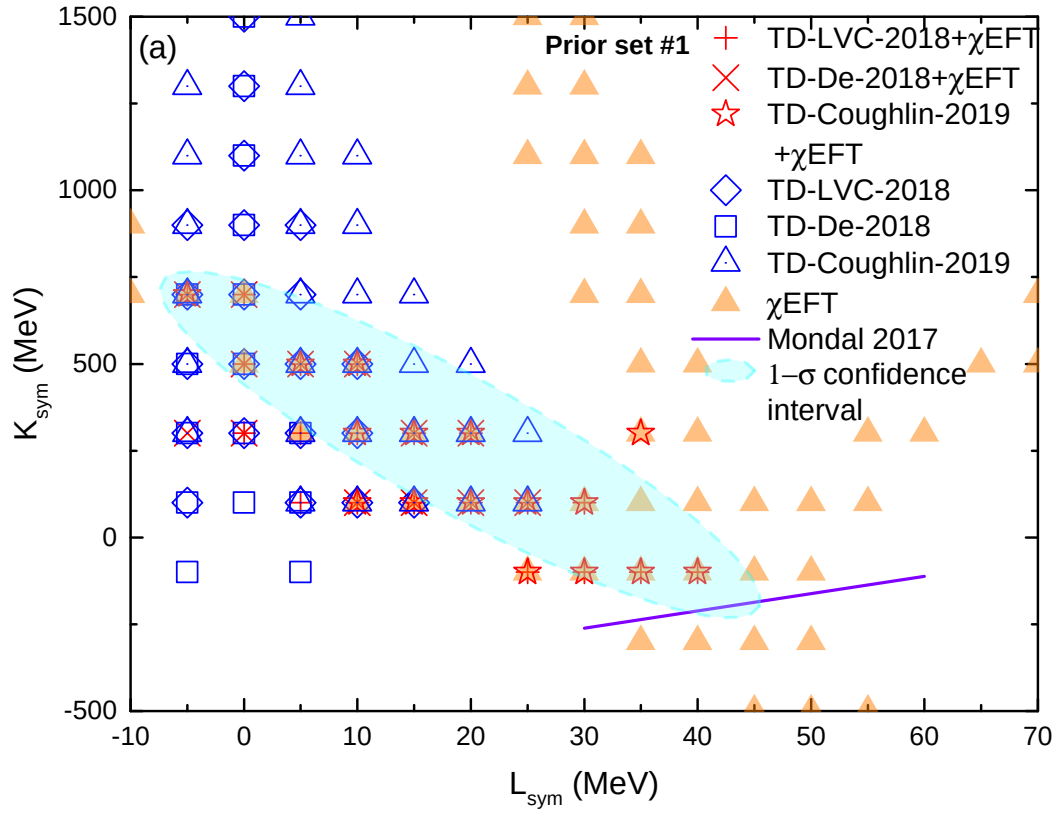


Figure 5.8 The values of the L_{sym} and K_{sym} inside of the 1- σ probability for the prior set #1 (a) and #2 (b) with the fit from Ref. [150]

Exploring a large set of RMF and Skyrme EDFs, the following relation $K_{\text{sym}} = \beta(3E_{\text{sym}} - L_{\text{sym}}) + \alpha$, with $\beta = -4.97 \pm 0.07$ and $\alpha = 66.80 \pm 2.14$ MeV, was suggested [150]. Fixing $E_{\text{sym}} = 32$ MeV (actually $E_{\text{sym}} = 32.1 \pm 0.3$ MeV is taken in Ref [150], but we keep fixed $E_{\text{sym}} = 32$ MeV in our analysis, for details see Table. 4.1 and related explanations), this correlation is shown in Fig. 5.8 with the legend Mondal 2017. This correlation was shown to originate from the physical condition that the energy per particle in NM should be zero at zero density [138]. Using the meta-model, the validity of this correlation has been confirmed and the contribution of higher order parameter (Q_{sym} , Q_{sat} , Z_{sym} and Z_{sat}) has also been investigated, adding about 200 MeV uncertainty to K_{sym} [138]. There is an overlap between the Mondal 2017 correlation line and the χ EFT favored domain, as expected (Fig. 5.8). However, the χ EFT favored domain is much larger since we have considered only the $n_0 \geq 0.12 \text{ fm}^{-3}$ energy band in NM. The constrain at very low density is thus not included in the χ EFT favored domain.

We have also analyzed the impact of the ISGMR constraints on the $L_{\text{sym}} - K_{\text{sym}}$ correlation, but since this is a correlation among isovector empirical parameter, there is no impact of the ISGMR constraint.

Finally, the blue contours in Figs. 5.8 represent the $1-\sigma$ ellipses including both the GW and χ EFT constraints together. This ellipse is only weakly dependent on the prior sets #1 and #2. We therefore propose a new correlation which reproduces the joint probability as,

$$K_{\text{sym}} = \alpha_1 L_{\text{sym}} + \beta_1, \quad (5.3)$$

where $\alpha_1 = -18.83^{+3.00}_{-2.00}$ and $\beta_1 = 616^{+140}_{-180}$ MeV.

5.4.2 The Correlation between K_{sat} and Q_{sat}

The second correlation we analyze here is the one between K_{sat} and Q_{sat} . The physical origin of this correlation is related to the ISGMR constraint reflected into the parameter M_c defined below saturation density at $n_c \approx 0.11 \text{ fm}^{-3}$ [33, 34]. Setting $n_0 = n_c$ in the isoscalar channel ($\delta = 0$) of the meta-model, one can obtain the following relation: $M_c \approx 4.6K_{\text{sat}} - 0.18Q_{\text{sat}} - 0.007Z_{\text{sat}}$ [138]. Fixing $M_c = 1050 \pm 100$ MeV, this relation induces a correlation between K_{sat} and Q_{sat} . However, a general analysis based on meta-model shows that this correlation is rather weak from the various EDFs, and the parameter Q_{sat} is yet unknown [138]. Since Q_{sat} can be constrained by the GW data, it is worth analyzing the correlation $K_{\text{sat}} - Q_{\text{sat}}$ under the influence of GWs.

In Figs. 5.9, the $K_{\text{sat}} - Q_{\text{sat}}$ correlations are shown for various constraints with a spurious correlation found for Skyrme and Gogny EDFs from Ref. [34] as the legend Khan

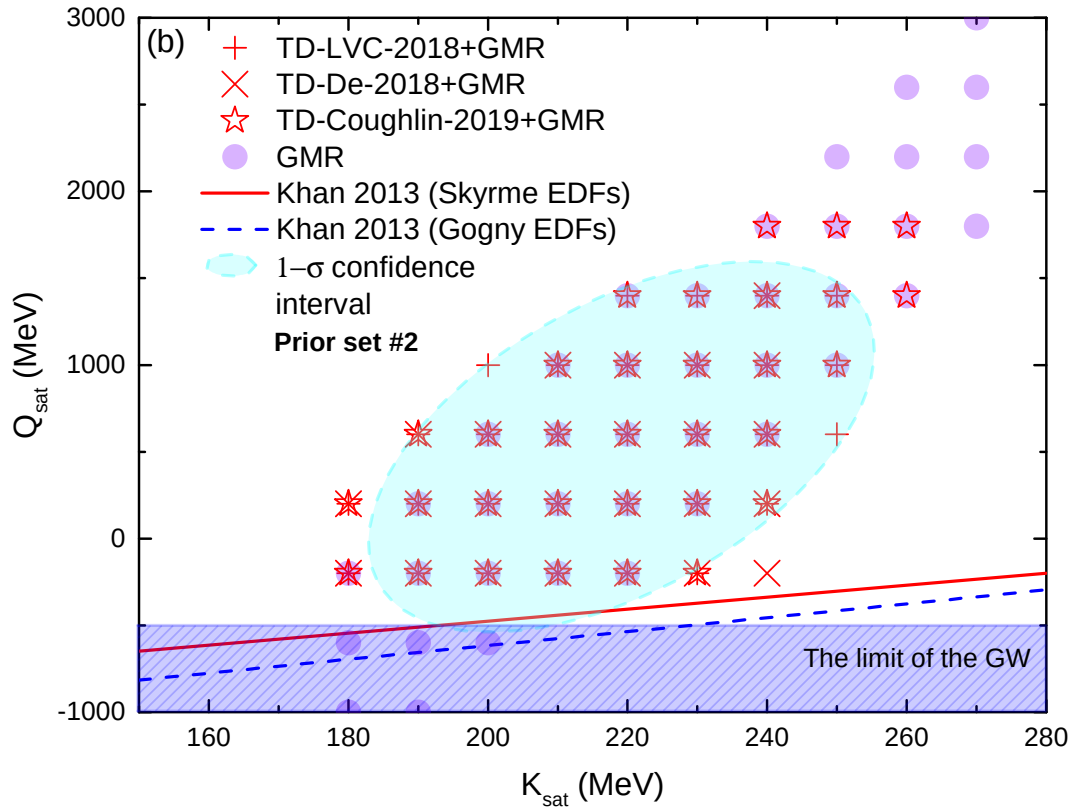
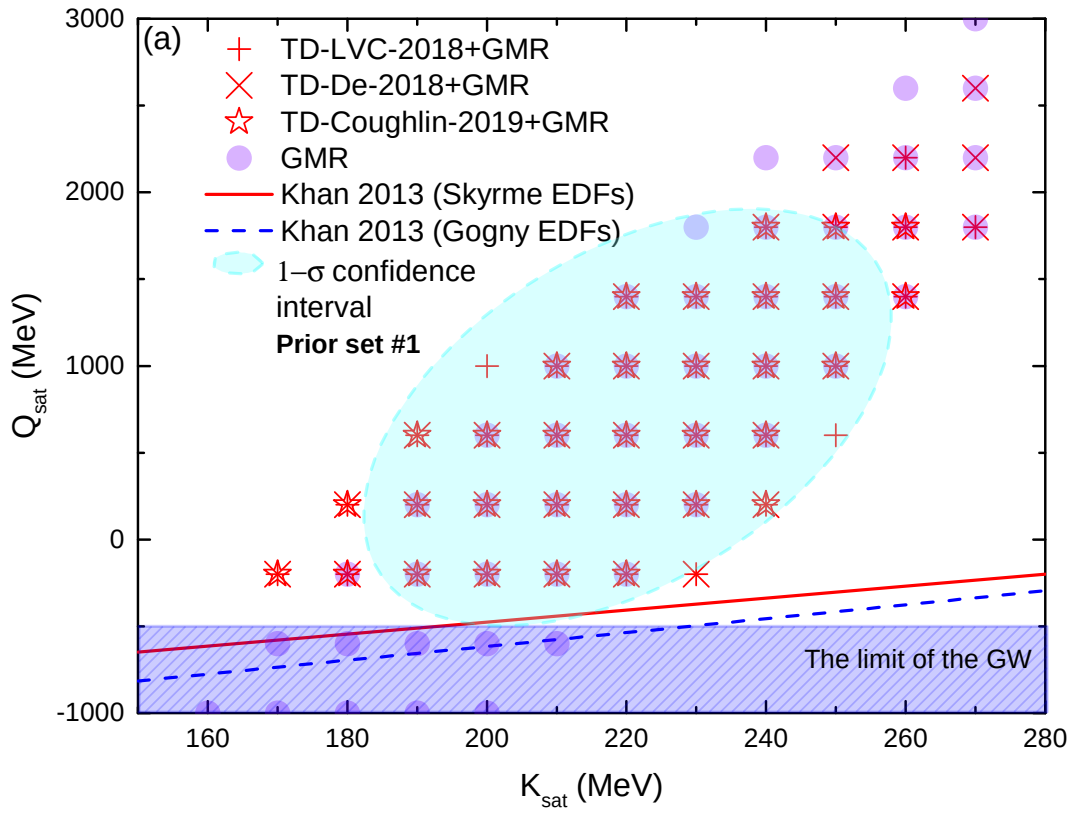


Figure 5.9 The values of the K_{sat} and Q_{sat} inside of the $1\text{-}\sigma$ probability for the prior set #1 (a) and #2 (b) with a spurious correlation found for Skyrme and Gogny EDFs from Ref [34]. Note that the χ EFT constraint is included for all joint posteriors

2013. The source of this correlation is the density dependent term from Skyrme and Gogny EDFs (see Ref. [34] for details). First, it should be stressed that the χ EFT constraint is included for all joint posteriors, but its effect was found negligible in this case. The domain allowed from the ISGMR constraint is shown with purple large dots, as previously discussed. A lower bound $Q_{\text{sat}} \geq -500$ MeV is shown, originating from the GW constraint as previously discussed in Fig. 5.3. Finally we represent the domain allowed by the GW data with the "+" (TD-LVC-2018), "x" (TD-De-2018) and "★" (TD-Coughlin-2019) symbols. There is a nice overlap between the GW data and the ISGMR. Furthermore, the confrontation of the GW data to the ISGMR correlation band allows to identify a smaller domain in $K_{\text{sat}} - Q_{\text{sat}}$, which is represented by the blue 1σ ellipse. However there is a discrepancy between the correlations from Skyrme and Gogny EDFs from Ref [34] and the GW, since the GW favors $-500 \leq Q_{\text{sat}} \leq 1500$ MeV and it forbids $Q_{\text{sat}} \leq -500$ MeV. The difference of this divergence can also be a hint for a phase transition.

From the 1σ confidence interval one can derive the following relation:

$$K_{\text{sat}} = \alpha_2 Q_{\text{sat}} + \beta_2, \quad (5.4)$$

where $\alpha_2 = 0.035^{+0.010}_{-0.010}$ and $\beta_2 = 199^{+20}_{-30}$. Furthermore, it seems that the ISGMR and GW constrain a different parameter at same time. While the GW is constraining Q_{sat} , the ISGMR impacts K_{sat} . Consequently, joint posteriors predict $170/180 \leq K_{\text{sat}} \leq 250/240$ MeV and $-500/-500 \leq Q_{\text{sat}} \leq 1200/1000$ MeV for the prior set #1/#2, respectively. An increased resolution of both constraints shall lead to more accurate determination of K_{sat} and Q_{sat} .

5.5 Concluding Remarks

In the present chapter, the main result is a marked tension between nuclear physics constraints (χ EFT and ISGMR) and the astrophysical constraints from GW170817 and $M_{\text{max}}^{\text{obs}}$, assuming the PDF from Refs. [26, 30]. The posteriors of GW for these two PDF favors $-4 \leq L_{\text{sym}} \leq 27$ MeV and $218 \leq K_{\text{sym}} \leq 648$ MeV while posteriors of χ EFT predicts $25 \leq L_{\text{sym}} \leq 49$ MeV and $-265 \leq K_{\text{sym}} \leq 608$ MeV. Consequently, the neutron star properties, $R_{1.4}$ and $P(2n_{\text{sat}})$, also exhibit marked tensions between these two constraints: the $p_{\tilde{\Lambda}}$ suggests smaller radii at $1.4M_{\odot}$ and higher pressure at $2n_{\text{sat}}$ at variance with nuclear physics. Since K_{sym} is responsible for the pressure at high density, but not for the radius at $1.4M_{\odot}$, this effect can be understood as a consequence of the low L_{sym} —high K_{sym} case. However the models of nuclear physics predict high L_{sym} —low K_{sym} , in an opposite way. This tension may be a hint for a quark phase

transition which would lower the radius of NS with masses larger than about $1.3M_{\odot}$, i.e. densities larger than about $2n_{\text{sat}}$. This conclusion should however be contrasted with the results obtained from a third analysis exploiting the multi-messenger signals from GW170817 (GW, EM and GRB), which favors a larger value of tidal deformability ($\tilde{\Lambda} \approx 600$). In this case, the tension with nuclear physics is solved and typical radii $R_{1.4} \approx 12.5 - 13$ km are obtained. Note that the multi-messenger analysis is based on the present state-of-the-art numerical simulations as well as on the single GW170817 event. Future improvements of the simulations as well as more binary neutron star events will potentially influence the result of the present Bayesian analysis. This illustrates the complexity of the multi-messenger analysis, which can be contrasted with the GW waveform analysis which is almost only based on general relativity theory and the assumption of low spin of the NS.

The second main result is that we could extract boundaries for a set of nuclear empirical parameters from the joint probability from TD-LVC-2018:

- (1) $L_{\text{sym}} = 0.00^{+12.00}_{-4.00} / 16.58^{+11.28}_{-6.79}$ MeV,
- (2) $K_{\text{sym}} = 438.57^{+210.12}_{-210.12}$ MeV (see related discussion for the prior set #2),
- (3) $Q_{\text{sat}} = -180^{+1222}_{-175} / -162^{+935}_{-175}$ MeV,
- (4) $Q_{\text{sym}} = -270^{+1690}_{-1126} / -169^{+1376}_{-748}$ MeV,

for the prior set #1/#2, respectively. It is also concluded that Q_{sat} is constrained by both GW and ISGMR analyses.

With the use of these nuclear parameters, we obtained the following values for the NS properties:

- (5) $R_{1.4} = 11.00^{+1.30}_{-0.25} / 10.98^{+1.90}_{-0.25}$ or $R_{1.4} = 12.00^{+0.30}_{-1.25} / 10.98^{+1.90}_{-0.25}$ km and
- (6) $P(2n_{\text{sat}}) = 24.61^{+24.42}_{-5.00} / 26.02^{+13.58}_{-5.00}$ MeV fm⁻³

for prior set #1/#2, respectively. It should be noted that these last results are in a good agreement with other recent analyses using GW [114, 120, 157].

Finally we have analyzed the $L_{\text{sym}}-K_{\text{sym}}$ and $K_{\text{sat}}-Q_{\text{sat}}$ correlations under the influence of GW170817, χ EFT and ISGMR constraints and proposed fits for our joint probability correlations. A different relation from nuclear physics is found: $K_{\text{sym}} = \alpha_1 L_{\text{sym}} + \beta_1$ where $\alpha_1 = -18.83^{+3.00}_{-2.00}$ and $\beta_1 = 616^{+140}_{-180}$ MeV. Another correlation which we found

is the $K_{\text{sat}} - Q_{\text{sat}}$ one. As one expects, this relation is highly correlated with the ISGMR [33, 34]. However we showed that GW has also a decisive role in determining these parameters since it constrains Q_{sat} . Consequently, we found the following relation: $K_{\text{sat}} = \alpha_2 Q_{\text{sat}} + \beta_2$ where $\alpha_2 = 0.035^{+0.010}_{-0.010}$ and $\beta_2 = 199^{+20}_{-30}$. All things considered, increasing the accuracy on the determination of tidal deformability from GW, as well as M_c from the ISGMR, will lead to a better determination of K_{sat} and Q_{sat} .

RESULTS AND DISCUSSIONS

In the present thesis, we have investigated the nuclear equation of state (EoS), impacting the structure of neutron stars (NS). On this purpose, we first discussed two different cases of the NS core: the phase transition to hypernuclear matter and the traditional nuclear matter approach.

Hypernuclear matter for NSs is problematic since hypernuclear equation of states decrease the pressure inside the NS core [9–13, 15] leading to smaller maximum mass than the observational maximum mass for NS ($2M_{\odot}$ [23, 24]). One of the solution to this "puzzle" could be the better understanding of the hyperon interaction inside the nuclear medium, which can be extracted from experiments on multi-strange hypernuclei. In this case superfluidity (or pairing) in hypernuclei could impact the interpretation of the experiments. Therefore, the Λ pairing channel was studied for multi-strange hypernuclei in the first part of the thesis. On this purpose, we considered Hartree-Fock-Bogoliubov (or Bogoliubov-de Gennes) formalism [35, 36] to evaluate ground state properties of $^{40}_{-\Lambda}\text{Ca}$, $^{132}_{-\Lambda}\text{Sn}$ and $^{208}_{-\Lambda}\text{Pb}$ hypernuclei which have closed proton and neutron shells, since the semi-magicity often guarantees that nuclei remain at, or close to, sphericity. We first investigated the possibility of $N\Lambda$ (N =proton or neutron) pairing channel by comparing the Fermi energies of each nucleons and Λ by using the Hartree-Fock approach. Since the energy difference between nucleons and Λ Fermi levels is usually large (more than 5 MeV) in the considered nuclei, the $N\Lambda$ pairing is quenched in most of the cases. We then fit a $\Lambda\Lambda$ pairing interaction into the Hartree-Fock-Bogoliubov equations, which magnitude is calibrated to be consistent with the maximum predictions for the Λ pairing gap in hypernuclear matter [73]. Generally, we found that $\Lambda\Lambda$ pairing could be active if the energy gap between orbitals is smaller than 3 MeV. Under this condition, Λ pairing could impact densities and binding energies. Since only a weak spin-orbit interaction is expected in the Λ channel, Λ states are highly degenerated and usually distant by more than 3 MeV in energy. Therefore, Λ -related pairing effect can usually be neglected in most of hypernuclei, except for hypernuclei which have a single particle gap lower than 3 MeV around the Fermi level.

With the advent of a first gravitational wave detection from a binary NS merger (GW170817) [25, 157], a new era for nuclear astrophysics has begun, since it has provided an additional observable related to the EoS of NS: Tidal deformability ($\tilde{\Lambda}$) [27–29]. It has also opened a possibility to test different EoSs for NSs. Taking advantage of GW constraints on tidal deformability, we tested the validity of the traditional nuclear matter hypothesis in the second part of the thesis. On this purpose, nuclear EoSs were generated by using observational data such as the maximum mass ($2M_{\odot}$ see Refs. [23, 24] for details) and $\tilde{\Lambda}$ constrained from the gravitational wave event of GW170817 [25, 26, 30, 31], as well as predictions from nuclear physics such as Chiral Effective Field Theory (χ EFT) [32] and Isoscalar Giant Monopole Resonance (ISGMR) [33, 34]. Our main results are the presence of marked tensions between various analyses of the GW signal from GW170817, depending on the inclusion or absence of multi-messenger additional constraints, and also marked tensions between astrophysical and nuclear physics constraints. For instance, the posteriors using the Ref. [30] (TD-De-2018) favors $L_{\text{sym}} = 0_{-3}^{+2}$ MeV, $K_{\text{sym}} = 390_{-400}^{+1110}$ MeV, while Ref. [31] (TD-Coughlin-2019) favours, $L_{\text{sym}} = 17_{-15}^{+15}$ MeV, $K_{\text{sym}} = 275_{-330}^{+890}$ MeV. The posterior predictions using $\tilde{\Lambda}$ from Ref. [26] (TD-LVC-2018) are intermediate between these two cases. This tension also exists for the radius predictions $R_{1.4}$, since $R_{1.4} = 10.7_{-0.3}^{+2.1}$ km in the case of TD-De-2018 and TD-LVC-2018, while it is $R_{1.4} = 13.1_{-0.5}^{+0.5}$ km in the case of TD-Coughlin-2019. These probability density functions (PDFs) of $\tilde{\Lambda}$ are however more consistent in their predictions for the pressure and we have found $P(2n_{\text{sat}}) = 45_{-25}^{+35}$ MeV fm $^{-3}$ for prior set #1 (see Sec. 4.8 for details). Besides, these predictions are also in marked tension with the posteriors obtained from χ EFT which predict $L_{\text{sym}} = 35_{-10}^{+7}$ MeV, $K_{\text{sym}} = 14_{-265}^{+600}$ MeV, $R_{1.4} = 13.0_{-1.2}^{+0.8}$ km and $P(2n_{\text{sat}}) = 12_{-4}^{+23}$ MeV fm $^{-3}$ for prior set #1. It is interesting to note that there is a marked tension in the values for L_{sym} between all $\tilde{\Lambda}$ -PDF analyses and the χ EFT one. However, it should be noted that for the radius $R_{1.4}$ the multi-messenger $\tilde{\Lambda}$ -PDF from Ref. [31], which is peaked at $\tilde{\Lambda} \approx 600$ is in good agreement with χ EFT predictions.

The tensions presented here between the posterior predictions are marked, but still consistent at $2\text{--}3\sigma$. The reduction of the uncertainties in our predictions requires a reduction of the observational or experimental uncertainties. Hence increasing the accuracy on the determination of tidal deformability from gravitational wave, as well as M_c from the ISGMR, will lead to a better determination of K_{sat} and Q_{sat} and NS properties. Increasing the number of gravitational wave signals of binary NS merger is also a way to refine our present analysis and conclude on the strength of the tension between multi-physics constraints. Ultimately including hyperon degree of freedom in the GW constrained EoS, shall allow to bridge the two main parts of the present work in a unified form.

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Publications From the Thesis

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Papers

1. H. Güven, K. Bozkurt, E. Khan, and J. Margueron, " $\Lambda\Lambda$ pairing in multi-strange hypernuclei", Physical Review C 98, 014318 (2018).
2. H. Güven, K. Bozkurt, E. Khan, and J. Margueron, "Multi-messenger and multi-physics Bayesian inference for GW170817 binary neutron star merger", Physical Review C 102, 015805 (2020).

Projects

1. Prof. Dr. Elias Khan, 1059B141700658, International Research Fellowship Programme for PhD Students 2214-A, Scientific and Technological Research Council of Turkey (TÜBİTAK), Doktora Bursiyeri, 17/10/2017
2. Prof. Dr. Kutsal Bozkurt, 118F098, The Scientific and Technological Research Projects Funding Program 1001, Scientific and Technological Research Council of Turkey (TÜBİTAK), Doktora Bursiyeri, 02/02/2019
3. Prof. Dr. Kutsal Bozkurt, FBI-2018-3325, İşbirliği Geliştirme, YTÜ-BAP, Doktora Araştırmacı, 02/05/2019